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Numerical Approaches to Traffic Flow Simulation: From Vehicle-Based Models to Continuum and Particle Methods

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ABSTRACT

Traffic-flow simulation employs a range of mathematical models and numerical approaches to represent vehicle dynamics, congestion propagation, and aggregate traffic behavior. However, the relationship between the mathematical representation of traffic and its numerical implementation is not always made explicit when different simulation approaches are compared. This paper reviews numerical approaches to traffic-flow simulation, covering microscopic vehicle-based models, cellular automata, kinetic formulations, macroscopic continuum models, and particle-based continuum methods. The review distinguishes traffic representation from numerical discretization and examines how these approaches differ in computational structure, spatial and temporal evolution, numerical properties, and geometric flexibility. Particular attention is given to particle-based methods, including Smoothed Particle Hydrodynamics (SPH), as alternative frameworks for discretizing continuum traffic models. Their potential advantages are considered alongside challenges involving numerical consistency, boundary treatment, computational cost, and the representation of traffic-specific dynamics. Rather than presuming the superiority of any particular method, the paper develops a comparative perspective for identifying candidate research opportunities and evaluating the suitability of alternative numerical formulations. The resulting framework motivates further investigation through mathematical analysis, controlled numerical experiments, and comparison with established traffic-flow models.

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1. INTRODUCTION

Traffic flow simulation is an important computational tool for analyzing vehicle dynamics, congestion formation, traffic propagation, and transportation networks. The complexity of traffic systems has led to the development of mathematical models at different levels of representation, ranging from individual vehicle behavior to aggregate traffic dynamics [1],[2],[3]. These models differ not only in their physical interpretation but also in how they must be represented and solved computationally.

Microscopic models describe traffic through individual vehicles and their interactions, commonly using variables such as position, velocity, and acceleration. Macroscopic models, in contrast, represent traffic through aggregate quantities such as density and flow. A classical example is the Lighthill-Whitham-Richards (LWR) model, which describes traffic evolution using a conservation equation and a relationship between flow and density [4],[5]. Between or alongside these representations, kinetic approaches describe traffic through distributions of vehicle states, while cellular automata represent traffic using discrete states and transition rules

[6],[7]. More recent studies have also explored hybrid microscopic–macroscopic formulations and alternative kinetic approaches to traffic modelling [8],[9],[10].

The mathematical representation of a traffic model is closely related to, but distinct from, the numerical method used to compute its evolution. Microscopic models generally require the time integration of vehicle-state equations, whereas macroscopic continuum models are commonly formulated as partial differential equations and solved using spatial discretization methods, including finite-volume schemes [1],[11]. The Cell Transmission Model (CTM), for example, provides a discrete computational formulation of macroscopic traffic flow that is consistent with the hydrodynamic theory of traffic and can be interpreted in relation to finite-volume methods for conservation laws [12],[13]. In this context, numerical fluxes and the treatment of discontinuities are important components of the computational scheme [11],[14].

Discrete traffic models require a different interpretation. Cellular-automata (CA) models use discrete spatial cells, vehicle states, and prescribed transition rules to define traffic evolution. The Nagel–Schreckenberg model is a well-known example in which vehicle movement is governed by discrete update rules [7]. Although both CA models and CTM employ spatial cells, their mathematical and computational roles are different: CA cells form part of the traffic representation itself, whereas CTM cells act as control volumes in the numerical discretization of an underlying macroscopic conservation model [12],[15]. Consequently, the use of spatial cells alone is not sufficient to classify two traffic-simulation approaches as belonging to the same modelling or numerical framework.

Recent research has extended the range of computational approaches beyond conventional fixed-grid discretization. Particle-based methods have been investigated as numerical approaches for approximating macroscopic vehicular traffic-flow models. In particular, Titakis et al. in [16] compared a particle method with alternative numerical methods for a macroscopic automated-vehicle traffic-flow model. In a related transportation domain, Yuan et al. in [17] developed a Smoothed Particle Hydrodynamics (SPH) framework for macroscopic pedestrian-flow simulation. These studies provide examples of moving computational particles being used to approximate transportation-flow models, although the traffic contexts and mathematical formulations differ. SPH itself is a mesh-free particle method based on kernel approximations, with foundational formulations developed in the context of computational astrophysics and fluid-related problems [18],[19],[20].

These developments motivate a broader computational perspective on traffic-flow simulation. Rather than classifying traffic models only according to their physical or behavioural scale, it is also useful to examine how their mathematical representations are converted into numerical representations and computational update procedures. From this perspective, microscopic, cellular-automata, kinetic, macroscopic continuum, and particle-based approaches involve different combinations of state representation, spatial discretization, temporal evolution, and computational elements.

This paper reviews numerical approaches to traffic-flow simulation from this computational perspective. The main question addressed is: how does the mathematical representation of traffic flow determine its numerical representation, and what computational characteristics distinguish the resulting approaches? The review focuses on the relationship between traffic representations and their numerical implementations, rather than providing a detailed mathematical analysis of model existence, uniqueness, or convergence. Particular attention is given to the distinction between Eulerian grid-based and Lagrangian particle-based representations, including the emerging use of particle methods and SPH in transportation-flow simulation.

The main contribution of this review is a computational taxonomy connecting traffic-flow representations with their corresponding numerical approaches. The discussion identifies the computational elements used by each approach, the ways in which spatial and temporal evolution are represented, and the principal advantages and limitations relevant to traffic simulation. Particle-based and mesh-free approaches are considered as potential alternatives for discretizing continuum traffic models, with attention to the numerical and traffic-specific challenges that must be addressed before their suitability can be established.

2. REVIEW METHODOLOGY

This paper adopts a focused literature review approach to examine numerical methods for traffic-flow simulation, with particular attention to the relationship between traffic representation and numerical discretization. The literature collection began with foundational and relevant studies identified during an initial investigation of traffic-flow modelling and numerical methods. This initial collection was subsequently expanded through iterative keyword searches.

The searches covered topics including traffic-flow modelling, microscopic traffic simulation, macroscopic traffic models, kinetic traffic theory, cellular automata, numerical discretization, particle-based methods, mesh-free methods, and Smoothed Particle Hydrodynamics (SPH). No strict publication-year restriction was imposed. However, greater emphasis was placed on studies published from 2020 onward to

capture recent developments, while earlier publications were included when they provided important theoretical or methodological foundations.

The identified publications were examined qualitatively based on their relevance to the scope of this review. Particular attention was given to their mathematical representations of traffic, numerical formulations, computational procedures, and reported advantages and limitations. The studies were then organized and compared according to their traffic representations and numerical approaches, providing the basis for the classification and discussion presented in the following sections.

The objective of this review is to provide a focused comparative synthesis of relevant approaches rather than an exhaustive systematic review of all published traffic-simulation methods. Accordingly, the discussion emphasizes the conceptual and computational relationships among different modelling and numerical approaches, with particular attention to the potential and challenges of particle-based methods in traffic-flow simulation.

3. NUMERICAL REPRESENTATIONS OF TRAFFIC FLOW

Traffic-flow simulation can be formulated using different representations of the traffic state. These representations differ in the entities or quantities used to describe traffic, the information retained by the model, and the mathematical variables governing their evolution. Common approaches include microscopic vehicle-based models, cellular automata, kinetic formulations, and macroscopic continuum models. Particle-based continuum methods provide a further computational representation in which continuous traffic quantities are approximated using moving numerical elements.

These approaches should not be interpreted simply as successive levels of model complexity. They are based on different modelling assumptions and retain different types of information. Microscopic models explicitly describe individual vehicles, kinetic models represent distributions of vehicle states, and macroscopic models describe aggregate traffic quantities. Cellular automata employ discrete states and transition rules, while particle-based continuum methods represent numerical approximations of continuous fields using computational particles.

An important distinction throughout this classification is the difference between a traffic representation and the numerical method used to compute its evolution. A traffic representation specifies the state variables and relationships used to describe the traffic system. A numerical method provides the computational procedure for advancing or approximating that representation. Consequently, similar computational structures do not necessarily imply equivalent mathematical models. For example, cellular automata and the Cell Transmission Model (CTM) both employ spatial cells, but their cells have different mathematical roles. In cellular automata, cells form part of the discrete traffic representation; in CTM, cells serve as control volumes for discretizing a macroscopic conservation model.

3.1 Microscopic Vehicle-Based Representation

Microscopic traffic models represent traffic through individual vehicles and their interactions. Each vehicle is typically described by state variables such as position, velocity, and acceleration. Its motion is determined by its current state and by interactions with other vehicles, particularly the preceding vehicle in car-following formulations.

A simplified vehicle representation can be expressed as:

$$\frac{dx_i}{dt} = v_i, \qquad \frac{dv_i}{dt} = a_i, \tag{1}$$

where x_i, v_i, and a_i denote the position, velocity, and acceleration of vehicle i, respectively. The acceleration is generally determined by a behavioural or car-following relationship, which may depend on the vehicle's velocity, relative velocity, and spacing to surrounding vehicles. Different car-following formulations therefore produce different vehicle-level dynamics [21].

The complete traffic state is represented by the collection of individual vehicle states. This representation retains detailed trajectory information and can accommodate differences in vehicle characteristics and driving behaviour. Aggregate phenomena, including congestion formation and traffic waves, emerge from the interactions among simulated vehicles rather than being directly represented through continuum variables.

From a computational perspective, microscopic models require the temporal integration of vehicle-state equations and the evaluation of interactions among vehicles. Their computational requirements consequently depend on the number of vehicles, the complexity of the behavioural relationships, and the interaction structure. This representation is particularly suitable when individual trajectories, vehicle behaviour, or local vehicle interactions are the primary objects of interest.

3.2 Discrete and Cellular-Automata Representation

Cellular-automata (CA) models represent traffic using discrete spatial locations, vehicle states, and transition rules. The traffic domain is divided into cells, and vehicle states evolve through prescribed rules at discrete time steps. The resulting computational state is inherently discrete.

A generic CA system can be represented as:

$$S^{n+1} = \mathcal{F}(S^n), \tag{2}$$

where S^n denotes the complete discrete traffic state at time step n, and $\mathcal{F}$ represents the collection of transition rules.

The Nagel–Schreckenberg model is a representative example of this approach. It describes vehicle movement through a sequence of discrete operations involving acceleration, deceleration, movement, and stochastic effects [7]. Subsequent studies have extended cellular-automata formulations to account for additional traffic characteristics, including driving-style differences and mixed traffic conditions [15],[22].

The defining characteristic of CA is not merely the use of spatial cells, but the use of discrete states and transition rules as the basis of the traffic model itself. This distinguishes CA from numerical methods that employ cells to approximate an underlying continuous model. Although both CA and CTM use discrete spatial locations, their mathematical interpretations differ. In CA, the discrete cell structure is part of the model representation; in CTM, the cells act as computational control volumes for a macroscopic conservation equation [12],[23].

CA models can generate complex collective traffic patterns from relatively simple local rules. Their discrete update structure can also be computationally convenient. However, their physical interpretation depends on the relationship between cell length, time-step duration, and the traffic quantities represented. The discretization of vehicle position and velocity may also influence the level of detail with which continuous traffic behaviour can be described.

3.3 Kinetic Representation

Kinetic models represent traffic through a distribution of vehicles over a microscopic state space rather than explicitly tracking every vehicle. A typical kinetic description uses a distribution function $f(x, v, t)$, which represents the distribution of vehicles at position x, velocity v, and time t.

This representation retains information about the distribution of vehicle states while avoiding the explicit representation of every individual vehicle. Aggregate quantities can subsequently be obtained from the distribution. For example, density may be obtained by integrating the distribution over velocity:

$$\rho(x, t) = \int f(x, v, t)\, dv. \tag{3}$$

Kinetic models therefore occupy an intermediate conceptual position between microscopic and macroscopic descriptions. They can retain information about velocity distributions and heterogeneous traffic states that is lost in a simple macroscopic representation, while representing traffic through a distribution rather than through a complete list of individual vehicles.

The term kinetic is used in several related but mathematically distinct traffic-modelling approaches. For example, kinetic theory formulations describe the evolution of vehicle-state distributions [6], whereas the Traffic Reaction Model employs a kinetic compartmental formulation [9]. Kinetic Monte Carlo methods, in contrast, use stochastic event-based procedures to simulate traffic evolution [10]. These approaches share a connection to kinetic descriptions or stochastic state evolution, but they should not be treated as identical numerical formulations.

The additional state dimension introduced by velocity or other microscopic variables also increases the dimensionality of the computational problem. The numerical representation must therefore account for both physical space and the relevant microscopic state space.

3.4 Macroscopic Continuum Representation

Macroscopic models represent traffic as a continuum through aggregate quantities such as density, flow, and mean velocity. Individual vehicles are not explicitly represented. Instead, traffic evolution is described through field variables that vary over space and time.

The classical Lighthill–Whitham–Richards framework, for example, is based on the conservation equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial q}{\partial x} = 0, \tag{4}$$

where ρ is traffic density and q is traffic flow. A constitutive relationship, such as a fundamental diagram, provides the relationship between traffic flow and density. The LWR formulation originates from the kinematic-wave descriptions developed by Lighthill and Whitham (1955) and Richards (1956) [4]. Its

conservation structure provides a basis for representing the propagation of changes in traffic density, including the formation and movement of traffic discontinuities.

Macroscopic models can also employ more detailed formulations involving additional state variables, such as mean velocity. These formulations extend the representation beyond a single conservation equation and can describe traffic dynamics that are not captured by the basic LWR model. The appropriate formulation depends on the traffic phenomena and level of detail required.

The defining characteristic of the macroscopic representation is the use of continuous traffic fields rather than individual vehicle states or discrete vehicle locations. This allows large traffic systems to be represented through aggregate variables without explicitly maintaining the trajectory of every vehicle. However, aggregation also means that individual vehicle behaviour and microscopic heterogeneity are not directly resolved.

It is important to distinguish the macroscopic model from the numerical method used to solve it. A conservation equation may be approximated using finite-difference, finite-volume, or other numerical schemes. The CTM, for example, provides a discrete computational formulation of macroscopic traffic flow based on spatial cells and traffic demand–supply relationships [12],[23]. In this case, the underlying traffic representation remains macroscopic even though its numerical solution is computed on a discrete spatial domain.

3.5 Particle-Based Continuum Representation

Particle-based continuum methods provide another way of representing a continuous traffic field. Instead of associating the continuum variables with fixed spatial locations, the numerical representation uses moving computational particles.

The important distinction is that these particles need not represent physical vehicles. A particle may instead be a numerical element associated with a portion or approximation of a continuous traffic field. Consequently, the trajectory of a computational particle does not necessarily correspond to the trajectory of an individual vehicle.

This distinction separates particle-based continuum methods from microscopic vehicle-based models. Both may use moving computational elements, but their meanings are different. The meaning differences can be seen on Figure 1.

Microscopic model	→	particle/element represents a vehicle
Particle-based continuum	→	particle represents a numerical approximation of a field

Figure 1. The different of microscopic and particle-based continuum.

Particle-based representations can therefore be understood as an alternative representation of continuum traffic rather than simply as another form of microscopic simulation.

Particle-based numerical approaches have been investigated for macroscopic vehicular traffic-flow models. Titakis et al., for example, compared a particle method with alternative numerical methods for approximating the solutions of a macroscopic automated-vehicle traffic-flow model [16]. This provides a direct example of computational particles being used to approximate a vehicular traffic continuum rather than representing individual vehicles.

A related development is the application of Smoothed Particle Hydrodynamics (SPH) to transportation-flow modelling. Yuan et al. developed an SPH-based approach for macroscopic pedestrian-flow simulation [17]. SPH approximates field quantities through kernel-based interactions among computational particles, with foundational formulations introduced by Gingold and Monaghan (1977) and Lucy (1977), and subsequently reviewed and developed by Monaghan (1992) [19],[20]. The pedestrian-flow application demonstrates the use of SPH in a transportation context, but it should be distinguished from vehicular traffic modelling because pedestrian and vehicle dynamics involve different behavioural and operational constraints.

Particle-based continuum methods can therefore be understood as an alternative numerical representation of continuum quantities rather than simply another form of microscopic simulation. Their computational characteristics depend on the particle approximation, the interaction or neighbourhood structure, and the treatment of boundaries. The use of moving particles may offer flexibility in representing spatially varying fields, but it does not by itself guarantee improved accuracy, efficiency, or suitability for traffic applications. These properties require evaluation for the particular mathematical model and numerical formulation.

3.6 Representation of Traffic Across Scales

The approaches discussed above retain different amounts and types of information about the traffic state. They can be viewed schematically as can be seen in Figure 2:

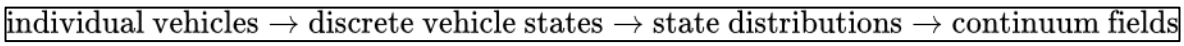

Figure 2. Representation of traffic across different scales.

with particle-based continuum methods providing an alternative numerical representation of the continuum rather than necessarily defining another physical scale. Microscopic models represent individual vehicle states; cellular automata use discrete locations, states, and transition rules; kinetic models describe distributions over microscopic state variables; and macroscopic models represent aggregate traffic quantities through continuous fields.

Particle-based continuum methods require a different classification. They do not necessarily define another physical scale of traffic representation. Instead, they provide a way of numerically representing and approximating a continuum model through moving computational elements. Thus, a particle-based continuum method may approximate a macroscopic traffic model without treating each computational particle as an individual vehicle.

The relationship among these approaches is therefore not a simple hierarchy from less detailed to more detailed models. Each representation retains different information and introduces different computational requirements. The choice of representation affects the mathematical formulation, while the numerical method determines how that formulation is approximated and advanced in time.

This distinction provides the basis for the following section, which examines the numerical discretization and computational procedures associated with the different traffic-flow representations.

4. NUMERICAL DISCRETIZATION AND COMPUTATIONAL METHODS

Once a traffic representation has been defined, it must be converted into a computational procedure. Continuous equations cannot be evaluated directly by a digital computer, and even discrete traffic models require a defined sequence of computational operations. Numerical simulation therefore involves approximating the mathematical representation using a finite computational structure and advancing that structure through time.

A general computational transformation can be written as can be seen in Figure 3:

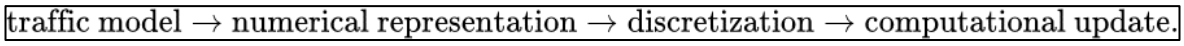

Figure 3. A general computational transformation for traffic model.

The specific form of this transformation depends on the representation being simulated. For vehicle-based models, the principal operation is temporal integration of vehicle-state equations. For continuum models, spatial and temporal discretization are required. For kinetic models, the physical and microscopic state dimensions must be discretized. For particle-based approaches, the continuum equations must be converted into relations between moving computational particles.

The distinction between these operations provides a more useful basis for comparing numerical methods than classification according to traffic-model type alone.

4.1 Temporal Discretization and Time Integration

Time discretization is required whenever a traffic model describes continuous evolution. Given a state at time t^n, a numerical method computes an approximation to the state at a subsequent time:

$$t^{n+1} = t^n + \Delta t. \tag{5}$$

For a vehicle-based model, an explicit Euler approximation gives:

$$\begin{aligned} x_i^{n+1} &= x_i^n + \Delta t\, v_i^n, \\ v_i^{n+1} &= v_i^n + \Delta t\, a_i^n, \end{aligned} \tag{6}$$

where t^n is a time at n-step, x_i^n, v_i^n, and a_i^n is a position, velocity, and acceleration of particle i at timestep n, respectively. More advanced time-integration schemes may be used to improve accuracy or stability. The important point is that temporal discretization converts continuous evolution into a sequence of computational states.

The explicit Euler method is presented here as a basic example rather than as a universally appropriate integration scheme. Other time-integration methods may be required depending on the governing equations, desired accuracy, and stability characteristics. The time step is therefore not merely an implementation parameter: it can influence numerical stability, accuracy, computational cost, and the physical interpretation of the simulated evolution.

For continuum and particle-based methods, temporal discretization is also required, although the state being advanced differs. A continuum solver advances field values associated with spatial locations or control volumes, whereas a particle method advances particle positions and the numerical quantities carried by those particles. In each case, the time-integration procedure must be compatible with the spatial approximation and governing equations.

4.2 Eulerian Spatial Discretization

For continuum traffic models, spatial discretization is required in addition to temporal discretization. In an Eulerian representation, the computational locations remain fixed while traffic quantities evolve through the domain.

For example, consider a one-dimensional conservation equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial q(\rho)}{\partial x} = 0, \tag{7}$$

where ρ denotes traffic density and $q(\rho)$ denotes the corresponding traffic-flow function. A finite-volume method divides the road into fixed control volumes. The density within each control volume is represented by a discrete quantity, and its evolution is determined by the fluxes crossing the boundaries:

$$\rho_i^{n+1} = \rho_i^n - \frac{\Delta t}{\Delta x}\left(F^n_{i+\frac{1}{2}} - F^n_{i-\frac{1}{2}}\right), \tag{8}$$

where $F^n_{i+\frac{1}{2}}$ and $F^n_{i-\frac{1}{2}}$ are numerical fluxes at the right and left interfaces of cell- i, respectively. The numerical flux F approximates the physical flow across a cell boundary and is determined by the selected numerical scheme. This formulation illustrates an important property of finite-volume methods: conservation is expressed directly through the balance of fluxes entering and leaving each control volume. The numerical flux formulation also plays an important role in approximating discontinuous solutions of conservation laws [11][14].

The Cell Transmission Model (CTM) provides an important traffic-flow example of this computational principle. It represents macroscopic traffic evolution through discrete road cells and traffic-flow transfers between them. Its formulation is consistent with the hydrodynamic theory of traffic and can be interpreted in relation to finite-volume discretization [12][23]. Thus, CTM illustrates how a macroscopic conservation model can be implemented through a discrete spatial structure.

The temporal and spatial resolutions must also satisfy appropriate numerical stability restrictions. For explicit schemes applied to hyperbolic conservation laws, a commonly used condition is the Courant–Friedrichs–Lewy (CFL) condition. For a scalar conservation law with flux function $q(\rho)$, a representative form is:

$$\frac{\Delta t}{\Delta x}\max_{\rho}\left|\frac{dq}{d\rho}\right| \leq C, \tag{9}$$

where C is a scheme-dependent stability bound, commonly not greater than one for standard explicit monotone schemes. The quantity $\frac{dq}{d\rho}$ represents the characteristic wave speed of the scalar conservation law. The precise admissible time step depends on the numerical scheme and the governing model [11].

4.3 State Space Discretization in Kinetic Models

Kinetic models require a more extensive numerical representation because their state variables include both physical position and microscopic variables such as velocity.

A distribution $f(x, v, t)$ therefore requires discretization over the relevant dimensions of x and v, followed by temporal evolution. The resulting computational problem may be represented as a collection of values $f^n_{i,k}$, where i identifies a spatial location, k identifies a velocity or state-space location, and n identifies the time level.

This additional state-space dimension allows kinetic models to retain information about the distribution of vehicle velocities, rather than representing traffic only through aggregate quantities such as density and mean velocity. However, it also increases the computational requirements because the numerical solution must resolve variations across multiple state dimensions. The resulting cost depends on the discretization resolution and the mathematical structure of the kinetic model [6].

It is important to distinguish this numerical treatment from stochastic event-based approaches such as kinetic Monte Carlo. Although both are associated with kinetic descriptions of traffic, their computational procedures are not necessarily equivalent. The discretization described here concerns the numerical representation of a distribution function, whereas kinetic Monte Carlo methods use stochastic event-based simulation procedures [10].

4.4 Eulerian and Lagrangian Numerical Representations

The spatial discretization approaches discussed above can also be distinguished according to whether their computational elements remain fixed or move with the represented system. This distinction is commonly expressed through the Eulerian and Lagrangian descriptions.

In an Eulerian representation, computational locations remain fixed in space. Traffic quantities such as density and flow are evaluated at these locations as the traffic state evolves through the domain. Grid-based finite-difference and finite-volume methods commonly follow this representation.

In a Lagrangian representation, computational elements move through the domain. Their positions evolve with the represented motion, and numerical information is associated with the moving elements. Microscopic vehicle simulation naturally follows this description because the computational entities correspond to physical vehicles whose positions and velocities change over time.

Particle-based continuum methods can also employ a Lagrangian representation. However, their particles have a different interpretation: they are numerical elements used to approximate a continuous field rather than necessarily representing physical vehicles. The distinction is therefore not simply between stationary and moving computational points, but also concerns the meaning of the elements used in the numerical representation.

This difference is important when comparing microscopic vehicle-based simulation with particle-based continuum methods. Although both may involve moving elements, their governing equations, state variables, and computational purposes are different. The former tracks physical vehicle states, whereas the latter approximates continuum quantities through moving numerical elements.

4.5 Particle Discretization of Continuum Models

A particle-based continuum method changes the spatial representation of a continuum model. Instead of storing the solution primarily at fixed grid locations, the solution is approximated using a set of moving computational particles .

The general idea can be expressed as can be seen in Figure 4:

$$\text{continuum field} \rightarrow \text{particles} \rightarrow \text{local particle approximation} \rightarrow \text{particle evolution.}$$

Figure 4. Particle discretization of continuum models.

A particle carries numerical information about the field, while neighboring particles provide the information needed to approximate local quantities and their spatial variation.

This approach avoids requiring a fixed background mesh for the spatial approximation. However, the absence of a mesh does not eliminate numerical discretization. It replaces one type of discretization with another: the continuous field must now be approximated from particle locations, particle properties, and interactions among neighboring particles.

Consequently, particle methods introduce their own numerical considerations, including particle distribution, neighbor identification, resolution, consistency, boundary treatment, and stability.

4.6 Kernel-Based Approximation and SPH

Smoothed Particle Hydrodynamics provides a specific framework for particle-based approximation. In SPH, a continuous field is reconstructed from neighboring particles using a smoothing kernel. A generic field quantity can be approximated as:

$$A(x) \approx \sum_j A_j W(x - x_j, h) V_j, \tag{10}$$

where W is the smoothing kernel, h is the smoothing length, and V_j represents the volume associated with particle j.

The SPH procedure therefore introduces an additional layer between the continuous field and the computational update can be seen on Figure 5:

$$\text{continuum equations} \rightarrow \text{particle discretization} \rightarrow \text{kernel approximation} \rightarrow \text{discrete equations} \rightarrow \text{particle update.}$$

Figure 5. The generic procedure of SPH approximation.

The significance of this procedure is that spatial derivatives and field quantities can be approximated using neighboring particles rather than values defined on a fixed mesh.

For traffic-flow simulation, however, the mathematical transfer cannot simply be assumed to follow the formulation used in fluid mechanics. The numerical particle representation must be connected to the traffic

variables and governing equations being modeled. Issues such as directionality, vehicle interactions, reaction effects, lane structure, and traffic-specific boundary conditions may require modifications to a conventional particle formulation.

At this stage, SPH should therefore be regarded as a numerical framework whose suitability for traffic remains an open question, rather than as an established solution to the limitations of existing traffic methods. This is consistent with the broader research objective of determining which properties of particle and mesh-free methods are genuinely useful for traffic simulation.

4.7 Numerical Properties of the Discretization

Different numerical representations introduce different numerical requirements. A useful comparison therefore requires more than identifying whether a method uses cells, vehicles, or particles. Important properties include:

- Conservation: whether quantities such as vehicle number are preserved appropriately by the discrete formulation.
- Consistency: whether the numerical approximation represents the underlying continuous or mathematical model correctly as resolution is refined.
- Stability: whether numerical errors remain bounded during simulation.
- Convergence: whether the numerical solution approaches the intended solution as spatial, temporal, or particle resolution increases.
- Resolution dependence: how strongly the result depends on cell size, time step, particle spacing, or kernel parameters.
- Boundary treatment: how road entrances, exits, lane boundaries, walls, and other boundaries are represented.
- Discontinuities: how the method handles sharp changes such as traffic shocks and congestion fronts.
- Computational cost: how the number of computational elements and operations scales with problem size.
- Geometric flexibility: how readily the method can accommodate multidimensional or complex geometries.
- Scalability: how computational performance changes as the spatial, temporal, or vehicle resolution increases.

These properties provide a common numerical vocabulary for comparing otherwise very different traffic simulation approaches.

5. COMPARISON OF NUMERICAL APPROACHES

The preceding sections have described the principal representations of traffic flow and the numerical procedures used to implement them. This section compares these approaches in terms of computational cost, numerical properties, and spatial flexibility. The objective is not to identify a universally superior method, but to examine the trade-offs that influence their suitability for different traffic-simulation problems.

5.1 Computational Cost and Representation

The computational cost of a traffic simulation is closely related to the number and type of computational elements involved. Microscopic models maintain individual vehicle states and evaluate interactions between vehicles. Their computational requirements therefore depend on the number of vehicles and the complexity of the interaction model. Cellular automata employ discrete states and rule-based updates, which can simplify computation, although their efficiency and realism depend on the selected rules and discretization [24].

Macroscopic models represent traffic through aggregate variables, allowing simulations to operate without explicitly tracking every vehicle. Their computational cost is consequently influenced by the spatial resolution, numerical scheme, and complexity of the governing equations. Kinetic models introduce an additional dimension through the velocity distribution function, potentially increasing the computational requirements associated with discretizing and evolving the traffic state.

Particle-based continuum methods employ moving computational particles to approximate continuous fields. Their computational cost depends on particle number, neighborhood identification, and the operations required to evaluate local quantities. Although such methods avoid the need for a fixed background mesh in their particle approximation, this does not necessarily make them less expensive than grid-based methods. The relative cost depends on the problem, numerical formulation, resolution, and implementation.

5.2 Numerical Properties and Accuracy

Numerical properties provide another basis for comparison. Conservation, consistency, stability, and convergence are important considerations when evaluating the reliability of a traffic simulation. Their achievement depends on the mathematical formulation and numerical scheme rather than on the model category alone [25].

Finite-volume methods formulate the evolution of conserved quantities through fluxes across control-volume boundaries. This structure provides a natural framework for enforcing discrete conservation and treating discontinuities in conservation-law models. However, numerical diffusion and the treatment of sharp traffic-density transitions remain relevant considerations. The resulting accuracy depends on the flux formulation, spatial resolution, and time integration.

Particle-based methods approximate local quantities using moving computational elements and, in SPH, kernel-based interpolation. Their numerical behavior depends on particle distribution, kernel properties, smoothing length, and boundary treatment. Inadequate particle consistency can affect the accuracy of approximated gradients and other spatial quantities. These issues are particularly relevant when extending particle methods from conventional fluid mechanics to traffic-flow models, where the governing relationships and behavioral constraints differ.

Consequently, numerical accuracy should not be inferred from whether a method uses a grid or particles. It must be evaluated through appropriate consistency and convergence studies, stability analysis, and comparison against analytical solutions or established numerical benchmarks where available.

5.3 Geometric Flexibility and Representation of Traffic Phenomena

The spatial representation also affects how different approaches accommodate traffic geometry and dynamic behavior. Grid-based methods provide a structured computational domain that can be effective for large-scale continuum simulations. However, representing complex boundaries or irregular geometries may require additional treatment, depending on the grid and numerical scheme. Microscopic models naturally retain individual vehicle trajectories, while cellular automata represent movement through predefined discrete locations and transition rules [26].

Moving-particle methods offer a different form of spatial representation because their computational elements can move with the represented field. This property may be useful for investigating multidimensional traffic configurations or domains in which a fixed-grid representation becomes inconvenient. Nevertheless, moving particles do not automatically resolve traffic-specific challenges, such as lane constraints, preferred direction of travel, vehicle interactions, and boundary conditions.

The ability to reproduce congestion waves, shock propagation, bottleneck effects, and stop-and-go traffic also depends on the governing model and its numerical treatment. A flexible spatial representation alone does not guarantee that these phenomena will be reproduced accurately. The model must capture the relevant traffic mechanisms, and the numerical method must preserve their essential behavior.

5.4 Comparative Summary

Table 1 summarizes the computational characteristics and principal numerical considerations of the approaches discussed above. The comparison highlights differences in computational structure and potential limitations; it should not be interpreted as a quantitative ranking of performance, since these characteristics depend on the specific formulation, implementation, and simulation problem.

Table 1. Comparative computational characteristics of numerical approaches to traffic-flow simulation.

Approach	Numerical representation	Main computational operation	Principal consideration
Microscopic vehicle-based	Individual vehicle states	Time integration	Vehicle interactions and computational cost
Cellular automata	Discrete cells and states	Rule-based updates	Transition rules and discretization
Kinetic	Discretized distribution function	Space- and velocity-space evolution	Higher-dimensional computation
Macroscopic continuum	Grid points or control volumes	Spatial discretization and state/flux updates	Conservation, diffusion, and discontinuities
Particle-based continuum	Moving numerical particles	Particle approximation and evolution	Particle consistency, neighbor search, and boundaries
SPH	Moving particles with kernel support	Kernel-based approximation and particle updates	Kernel consistency, stability, and traffic-specific adaptation

The comparison indicates that each approach involves distinct trade-offs. Grid-based continuum methods provide established frameworks for solving conservation equations, whereas particle-based continuum methods offer an alternative spatial discretization using moving computational elements. However, the absence of a fixed background mesh does not, by itself, establish superior accuracy, efficiency, or geometric flexibility. These properties require evaluation under relevant traffic conditions. The next section therefore examines the research gaps that may motivate further investigation of particle-based methods.

6. RESEARCH GAPS AND OPPORTUNITIES FOR PARTICLE-BASED METHODS

The comparison of numerical approaches in the preceding section indicates that traffic-flow simulation involves trade-offs between computational efficiency, numerical accuracy, spatial representation, and the level of traffic detail retained. Although established grid-based and vehicle-based methods provide important foundations for traffic simulation, alternative numerical representations may offer opportunities to address specific limitations. Particle-based methods are one such possibility, particularly for continuum formulations in which the computational elements move independently of a fixed background mesh.

However, identifying a research opportunity requires distinguishing between a limitation of an existing traffic model, a limitation of its numerical discretization, and a limitation arising from the physical or behavioral assumptions of the model itself. A particle-based formulation may address some numerical or geometric challenges without resolving deficiencies in the underlying traffic model. The following subsections identify candidate areas for further investigation.

6.1 Numerical Consistency and Accuracy

One potential research direction concerns the accuracy and consistency of particle-based approximations when applied to traffic-flow equations. In SPH, continuous field quantities and their spatial derivatives are approximated using neighboring particles and smoothing kernels. The quality of these approximations depends on factors such as particle distribution, kernel properties, smoothing length, and boundary treatment.

These considerations raise questions about whether particle-based discretizations can reproduce the essential mathematical properties of traffic-flow models with sufficient accuracy. For example, a particle formulation derived from a continuum conservation law must be examined for its ability to preserve mass and represent sharp transitions in traffic density. Its behavior under changes in particle resolution and spatial distribution must also be assessed.

A relevant research opportunity is therefore to determine which particle formulations provide consistent and stable approximations of traffic-flow equations, and under what conditions. Such an investigation should compare particle-based methods with established numerical schemes rather than assume that a mesh-free formulation automatically provides greater accuracy.

6.2 Traffic-Specific Dynamics and Constraints

Traffic flow differs from conventional fluid motion in several important respects. Vehicles generally move in preferred directions, interact according to behavioral and safety constraints, and operate within lane structures and roadway boundaries. Traffic dynamics may also involve heterogeneous vehicle characteristics, finite acceleration and deceleration, and responses to changing traffic conditions.

Consequently, directly transferring a particle formulation developed for fluid mechanics to traffic simulation may not adequately represent the relevant traffic mechanisms. The mathematical formulation must distinguish the approximation of continuous traffic fields from the behavioral relationships that govern vehicle movement.

An opportunity for further research is to investigate how traffic-specific properties can be incorporated into particle-based continuum models without compromising their numerical consistency. This includes examining directional interactions, roadway boundaries, and the representation of congestion propagation. The objective should be to identify the minimum modifications required for a particle formulation to reproduce the selected traffic phenomena reliably.

6.3 Spatial Flexibility and Boundary Treatment

Particle-based methods represent a potentially useful alternative when the computational domain has complex geometry or when a fixed-grid representation becomes inconvenient. Moving computational elements may provide flexibility in representing spatially varying traffic states and multidimensional configurations.

Nevertheless, this potential advantage requires careful qualification. Particle methods still need mechanisms for representing roadway boundaries, lane constraints, inflow and outflow conditions, and

interactions with obstacles or changes in roadway geometry. Irregular particle distributions near boundaries may also affect the accuracy of local approximations.

Further investigation could therefore examine whether particle-based methods offer practical benefits for selected traffic geometries compared with established grid-based approaches. Such comparisons should consider not only geometric flexibility but also numerical accuracy, implementation complexity, and computational cost.

6.4 Validation and Comparative Evaluation

A further research need concerns the systematic evaluation of particle-based traffic models. Demonstrating that a numerical formulation can generate plausible traffic patterns is insufficient to establish its reliability or practical value. A model should be evaluated against known mathematical behavior, established traffic models, or appropriate empirical observations.

For particle-based methods, a validation framework should examine conservation properties, numerical stability, resolution dependence, and the reproduction of canonical traffic phenomena. Relevant examples include free-flow propagation, congestion-wave formation, shock propagation, bottleneck effects, and stop-and-go traffic. Comparisons should use equivalent initial conditions and, where possible, consistent boundary conditions and evaluation criteria.

Computational performance should also be considered. A particle method that offers greater spatial flexibility may nevertheless require more computation because of neighborhood searches, particle interactions, or additional numerical corrections. A meaningful comparison must therefore evaluate the trade-off between accuracy, computational expense, and the specific requirements of the traffic-simulation problem.

6.5 Research Opportunities and Implications

The candidate research directions identified above suggest that the principal question is not whether particle-based methods can replace existing numerical approaches universally, but whether they can provide demonstrable advantages for particular traffic-flow problems.

Three broad questions follow from this discussion:

1. **Numerical formulation**: Can a particle-based discretization represent the governing traffic-flow equations consistently while maintaining the relevant conservation and stability properties?
2. **Traffic-specific adaptation**: What modifications are required to account for directional movement, vehicle interactions, roadway boundaries, and characteristic congestion phenomena?
3. **Comparative performance**: Under what conditions, if any, do particle-based methods provide meaningful benefits over established grid-based or vehicle-based approaches in accuracy, computational cost, or geometric flexibility?

These questions provide a basis for a focused research program. An initial investigation could begin with a simple one-dimensional traffic-flow model, allowing the particle discretization to be evaluated against an established numerical reference before introducing more complex geometries or traffic behavior. This staged approach would help separate numerical errors from limitations of the underlying traffic model.

Ultimately, the value of particle-based methods must be established through mathematical analysis and reproducible numerical experiments. If their advantages are limited, or if their computational cost and numerical difficulties outweigh the benefits for a particular application, that finding would also provide a meaningful contribution to the assessment of numerical approaches for traffic-flow simulation.

7. DISCUSSION

The comparison presented in this paper highlights the close relationship between the mathematical representation of traffic flow and the numerical procedures used to simulate it. Microscopic, cellular-automata, kinetic, and macroscopic continuum approaches represent traffic through different computational structures, each retaining particular information about traffic states and dynamics. Their numerical implementations consequently involve different requirements for spatial and temporal discretization, state evolution, and boundary treatment. Selecting an appropriate approach therefore requires consideration of both the traffic phenomena to be represented and the numerical properties needed for reliable simulation.

An important implication is that the distinction between traffic models and numerical methods should be maintained when evaluating alternative approaches. A limitation associated with a particular numerical scheme does not necessarily imply a limitation of the underlying traffic model. Similarly, changing the numerical representation does not automatically resolve deficiencies in the governing equations or behavioral assumptions. This distinction is particularly relevant when investigating particle-based methods, because their potential advantages must be assessed separately from the suitability of the traffic model being discretized.

Particle-based continuum methods provide an alternative to fixed-grid discretization by representing continuous fields through moving computational elements. In principle, this representation may offer useful

flexibility for selected spatial configurations. However, the absence of a fixed background mesh is not sufficient evidence of improved numerical accuracy, computational efficiency, or physical realism. Particle consistency, neighborhood interactions, boundary conditions, and computational cost remain important considerations. Moreover, the behavioral and directional characteristics of traffic require appropriate mathematical treatment rather than direct reliance on formulations developed for conventional fluid mechanics.

These observations suggest that further investigation of particle-based traffic simulation should proceed through a staged evaluation. Initial studies should employ a clearly defined traffic-flow model and a simple computational domain, allowing the numerical formulation to be examined independently of additional complexities. The resulting method can then be evaluated against established numerical references using common initial and boundary conditions. Conservation, stability, convergence, resolution dependence, and computational cost should be assessed alongside the reproduction of relevant traffic phenomena. More complex configurations should be introduced only after the basic formulation has demonstrated acceptable numerical behavior.

The scope of this paper also imposes limitations on the conclusions that can be drawn. The comparison is qualitative and does not establish a universal ranking of numerical approaches. Computational performance depends on the specific formulation, implementation, problem size, and required resolution. Furthermore, the candidate research gaps discussed in Section 5 require systematic examination of the existing literature before they can be regarded as established gaps or novel research contributions. Detailed mathematical analysis and controlled numerical experiments are necessary to determine whether particle-based methods offer meaningful advantages for particular traffic-simulation problems.

Overall, the findings support a problem-driven approach to numerical traffic modelling. Rather than assuming that one representation is universally preferable, researchers should identify the requirements of the target problem, select a mathematically appropriate model, and evaluate candidate numerical methods against explicit criteria. Within this framework, particle-based and SPH-inspired methods merit investigation as possible alternatives, but their scientific value will depend on demonstrated numerical properties and comparative performance.

8. CONCLUSIONS

This paper has examined numerical approaches to traffic-flow simulation, focusing on the relationship between traffic-flow representations and their computational implementations. Microscopic vehicle-based models, cellular automata, kinetic models, and macroscopic continuum formulations represent traffic through different state variables and computational structures. Their numerical implementations consequently involve distinct approaches to time integration, spatial discretization, state evolution, and the treatment of computational boundaries.

The comparison highlights that the suitability of a numerical approach depends on the characteristics of the traffic problem, the properties of the underlying mathematical model, and the computational requirements of the simulation. No single approach can be considered universally superior based on its representation alone. Numerical accuracy, stability, conservation, computational cost, and geometric flexibility must be evaluated in relation to the intended application.

Particle-based continuum methods, including Smoothed Particle Hydrodynamics (SPH), provide an alternative framework for discretizing continuous traffic-flow models through moving computational elements. However, their potential benefits must be balanced against challenges involving particle consistency, boundary treatment, computational efficiency, and the representation of traffic-specific dynamics. Their suitability for traffic simulation therefore requires systematic mathematical analysis and comparison with established numerical methods.

Future research should establish a more detailed evidence-based assessment of existing particle-based traffic models and identify specific numerical or modelling limitations that remain unresolved. Subsequent investigations can then formulate and implement candidate particle-based methods, beginning with simple traffic-flow configurations and progressing through controlled validation and comparative evaluation. This approach would allow the advantages and limitations of particle-based methods to be assessed reproducibly, without presupposing their superiority over existing alternatives.

In conclusion, the development of numerical methods for traffic-flow simulation should remain problem-driven and evidence-based. Particle-based and SPH-inspired approaches represent promising subjects for investigation, but their scientific contribution must ultimately be demonstrated through mathematical consistency, numerical validation, and measurable performance relative to established methods.

REFERENCES

[1] P. Goatin, “Macroscopic traffic flow modelling: from kinematic waves to autonomous vehicles,” *Communications in Applied and Industrial Mathematics*, vol. 14, no. 1, pp. 1–16, Jan. 2023, doi: 10.2478/caim-2023-0001.

[2] Y. Wang *et al.*, "Macroscopic traffic flow modelling of large-scale freeway networks with field data verification: State-of-the-art review, benchmarking framework, and case studies using METANET," *Transp. Res. Part C Emerg. Technol.*, vol. 145, p. 103904, Dec. 2022, doi: 10.1016/j.trc.2022.103904.
[3] A. Mustapha, A. M. Abdul-Rani, N. Saad, and M. Mustapha, "Advancements in traffic simulation for enhanced road safety: A review," *Simul. Model. Pract. Theory*, vol. 137, p. 103017, Dec. 2024, doi: 10.1016/j.simpat.2024.103017.
[4] M. J. Lighthill and G. B. Whitham, "On kinematic waves. II. A theory of traffic flow on long crowded roads," *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences*, vol. 229, no. 1178, pp. 317–345, 1955. DOI: 10.1098/rspa.1955.0089.
[5] P. I. Richards, "Shock waves on the highway," *Operations Research*, vol. 4, no. 1, pp. 42–51, 1956. DOI: 10.1287/opre.4.1.42.
[6] L. Fermo and A. Tosin, "A fully-discrete-state kinetic theory approach to traffic flow on road networks," *Mathematical Models and Methods in Applied Sciences*, vol. 25, no.3, 2015, doi: 10.1142/S0218202515400023.
[7] K. Nagel and M. Schreckenberg, "A cellular automaton model for freeway traffic," *Journal de Physique I*, vol. 2, no. 12, 1992, doi: 10.1051/jp1:1992277.
[8] Y. Imai, H. Fujii, K. Okano, M. Matsudaira, and T. Suzuki, "Development of Dynamic Micro- and Macroscopic Hybrid Model for Efficient Highway Traffic Simulation: Model Extension to Merging Sections and Validation with Probe Data," *International Journal of Intelligent Transportation Systems Research*, vol. 22, no. 1, pp. 159–170, Apr. 2024, doi: 10.1007/s13177-024-00386-4.
[9] M. Pereira, B. Kulcsár, Gy. Lipták, M. Kovács, and G. Szederkényi, "The Traffic Reaction Model: A kinetic compartmental approach to road traffic modeling," *Transp. Res. Part C Emerg. Technol.*, vol. 158, p. 104435, Jan. 2024, doi: 10.1016/j.trc.2023.104435.
[10] Y. Sun, "Kinetic Monte Carlo simulations of 1D and 2D traffic flows: Nonlocal models with generalized look-ahead rules," *Transportation Research Part B: Methodological*, vol. 190, p. 103083, Dec. 2024, doi: 10.1016/j.trb.2024.103083.
[11] R. J. LeVeque, *Finite Volume Methods for Hyperbolic Problems*. Cambridge: Cambridge University Press, 2002. DOI: 10.1017/CBO9780511791253.
[12] C. F. Daganzo, "The cell transmission model: A dynamic representation of highway traffic consistent with the hydrodynamic theory," *Transportation Research Part B: Methodological*, vol. 28, no. 4, pp. 269–287, 1994. DOI: 10.1016/0191-2615(94)90002-7.
[13] A. S. Maulana and S. R. Pudjaprasetya, "Analysis of cell transmission model for traffic flow simulation with application to network traffic," *ANZIAM Journal*, vol. 63, no. 1, pp. 84–99, Jan. 2021, doi: 10.1017/S1446181121000080.
[14] S. K. Godunov, "A difference method for numerical calculation of discontinuous solutions of the equations of hydrodynamics," *Matematicheskii Sbornik*, vol. 47(89), no. 3, pp. 271–306, 1959.
[15] T. Feng, K. Liu, and C. Liang, "An Improved Cellular Automata Traffic Flow Model Considering Driving Styles," Sustainability, vol. 15, no. 2, p. 952, Jan. 2023, doi: 10.3390/su15020952.
[16] G. Titakis, I. Karafyllis, D. Theodosis, I. Papamichail, and M. Papageorgiou, "A comparative study of numerical methods for approximating the solutions of a macroscopic automated-vehicle traffic flow model," *Computers & Mathematics with Applications*, vol. 176, pp. 469–490, Dec. 2024, doi: 10.1016/j.camwa.2024.11.007.
[17] Y. Yuan, B. Goñi-Ros, H. H. Bui, W. Daamen, H. L. Vu, and S. P. Hoogendoorn, "Macroscopic pedestrian flow simulation using Smoothed Particle Hydrodynamics (SPH)," *Transp. Res. Part C Emerg. Technol.*, vol. 111, pp. 334–351, Feb. 2020, doi: 10.1016/j.trc.2019.12.017.
[18] L. B. Lucy, "A numerical approach to the testing of the fission hypothesis," *The Astronomical Journal*, vol. 82, pp. 1013–1024, 1977. DOI: 10.1086/112164.
[19] R. A. Gingold and J. J. Monaghan, "Smoothed particle hydrodynamics: Theory and application to non-spherical stars," *Monthly Notices of the Royal Astronomical Society*, vol. 181, no. 3, pp. 375–389, 1977. DOI: 10.1093/mnras/181.3.375.
[20] J. J. Monaghan, "Smoothed particle hydrodynamics," *Annual Review of Astronomy and Astrophysics*, vol. 30, pp. 543–574, 1992. DOI: 10.1146/annurev.aa.30.090192.002551.
[21] R. B. Ramachandra, B. Ghosh, V. Pakrashi, S. Moutari, and T. O. Alakoya, "A Comparative Study of Non-Linear Car Following Models in Real-Driving Scenarios," *IET Intelligent Transport Systems*, vol. 19, no. 1, Jan. 2025, doi: 10.1049/itr2.70098.
[22] Y. Jiang, Z. Yi, G. Xiao, H. Li, and Z. Yao, "Modeling the Effect of the Platoon Size of CAVs on Mixed Traffic Flow: A Cellular Automaton Method," *J. Adv. Transp.*, vol. 2023, no. 1, Jan. 2023, doi: 10.1155/2023/2761858.
[23] A. S. Maulana and S. R. Pudjaprasetya, "Analysis of cell transmission model for traffic flow simulation with application to network traffic," *ANZIAM Journal*, vol. 63, no. 1, pp. 84–99, Jan. 2021, doi: 10.1017/S1446181121000080.
[24] A. Rigler and G. Turk, "The Influence of Different Driving Parameters on Traffic with a Cellular Automata Model," *Applied Sciences (Switzerland)*, vol. 15, no. 23, Dec. 2025, doi: 10.3390/app152312404.
[25] A. Aggarwal, H. Holden, and G. Vaidya, "On the accuracy of the finite volume approximations to nonlocal conservation laws," *Numer. Math. (Heidelb).*, vol. 156, no. 1, pp. 237–271, Feb. 2024, doi: 10.1007/s00211-023-01388-2.
[26] L. Sica, F. Deflorio, M. Ferraro, and G. Calcagno, "Setting Key Model Parameters for Microscopic Traffic Simulation Using Vehicle Trajectory Data," *Sustainability*, vol. 18, no. 14, p. 7210, Jul. 2026, doi: 10.3390/su18147210.