



# A Comparison of The Predictive Ability between Logistic and Gompertz Model on COVID-19 Outbreak

Tita Haryanti <sup>a,\*</sup>, Brian Pamukti <sup>b</sup>

<sup>a</sup> School of Applied Science, Telkom University, Bandung, Indonesia

<sup>b</sup> School of Electrical Engineering, Telkom University, Bandung, Indonesia  
[tharyanti@telkomuniversity.ac.id](mailto:tharyanti@telkomuniversity.ac.id), [brianp@telkomuniversity.ac.id](mailto:brianp@telkomuniversity.ac.id)

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## ABSTRACT

A predictive model can be learned using historical information. Thereafter, information about a running case is combined with a predictive model to estimate the case's remaining flow time. The predictive model is based on data from past events, which can be used to make predictions for current operating situations. For example, the case of coronavirus disease 2019 (COVID-19), which is currently infecting the whole world, including Indonesia, have influenced various aspects, ranging from the educational environment, business, economy, to the companies. Data scientists are urgently needed who can help organizations improve their operational processes. Therefore, this journal discusses the prediction of the peak number of COVID-19 cases in Indonesia, using two prediction models, logistic and Gompertz. The results obtained show that the Gompertz model has higher accuracy than the logistic model, with an accuracy of 99.85%. This journal's results are expected to help organizations estimate the time to rebuild themselves after being affected by COVID-19.

\* Corresponding author at:

School of Applied Science, Telkom University  
Jl. Telekomunikasi No. 1, Terusan Buah Batu, Bandung, 40257  
Indonesia  
E-mail address: [tharyanti@telkomuniversity.ac.id](mailto:tharyanti@telkomuniversity.ac.id)

ORCID ID:

- First Author: 0000-0002-3631-2597
- Second Author: 0000-0003-3824-9306

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## 1. Introduction

The interdisciplinary area of data science is which aims at translating true value data. Data can be small or large, static or streaming, structured or unstructured. In the form of forecasts, computerized decisions, data-finding models, or some kind of insight-providing visualization of documents, the value can be given.

The interest in data science is rapidly on the rise. Recent years have seen the rise of data science as a modern and significant discipline. It can be seen as a combination of traditional disciplines such as analytics, data processing, databases, and distributed systems. New strategies need to be integrated to transform readily available data into meaning for individuals, organizations, and society. Many see data science as a future profession—the hype around big data and predictive analytics show this. Data ("Small" or "Large") is valuable to individuals and organizations and can only increase their significance [1].

Most of the digital universe information is unstructured, and companies have difficulty managing such vast volumes of data. The extraction of knowledge and meaning from data contained in there is the IS. It is one of the key challenges facing today's organizations. To detect patterns and forecast future trends and outcomes, predictive analytics is the method of extracting data from established data sets. New approaches to mining and learning are applied to generate predictions in an enterprise environment. Business analysis and market intelligence help with predictive analysis. Predictive analytics uses historical data to make predictions [1].

Models with a high predictive value can only be generated by experienced designers and analysts and can be used as a starting point for re-implementation or redesign. Hence, we promote the use of data from cases. Process mining permits the extraction of fact-based models. On the one side, it can be seen that all manner of inefficiencies exists. On the other hand, process mining may also imagine some workers' exceptional versatility in solving problems and varying workloads [1].

The findings from data mining can be descriptive as well as predictive. Decision trees, rules of the association, regression functions say something about the set of data used to learn the model [1] [2] [3]. Nevertheless, they can also be used to make projections for new cases, such as forecasting the peak number of COVID-19 cases in Indonesia based on the number of the first cases.

Based on the facts, the number of COVID-19 cases in Indonesia has increased day by day since March 2020. Not only does it have an impact on health that leads to death, but it also impacts on various aspects, such as the educational environment, business, economy, and even large companies, which implement work from home (WFH) and even layoffs. Therefore, based on these conditions, it becomes essential for researchers, especially in data science, to predict the peak number of COVID-19 cases to help organizations estimate the time to rebuild themselves after being affected by COVID-19.

In data science, many prediction models are often used to predict a case. One of them is the logistic model and the Gompertz model that is used in this journal. Various studies have used growth analyses to describe species growth [4] [5] [6] [7] [8], to compare different biometric growth trends within a species [4] [9], comparing one biometric to another [10] [11], or relate environmental conditions to energy expenditure and growth [12] [13] [14]. Given the substantial number of studies [15] [16] [17], There was no single standardized method for describing or evaluating the growth of body mass or other biometric measurements over time. [18] The system of Ricklefs does not require regression and is not discussed here.

One of the most widely used sigmoid models for growth data and other data is the Gompertz model [19], probably second only to the logistic model (also called the Verhulst model). Researchers suit the Gompertz model with everything from plant growth, bird growth, fish growth, and another animal growth to tumor growth and bacterial growth [20] [21] [22] [23] [24] [25] [26] [27] [28] [29], and the literature is massive. The Gompertz belongs to the Richards family of triparameter sigmoidal growth models, in addition to traditional models such as the negative exponential (including the Brody), the logistic, and the von Bertalanffy (or Bertalanffy alone) [30] [31]. The literature contains numerous Gompertz model parameterizations and reparameterizations. However, no systematic review has attempted these and their properties.

The cumulative Gompertz-Logistic model also rapidly became a favorite from the 1920s onwards in fields other than human mortality, such as predicting increased demand for goods and services, cigarette sales, growth in rail traffic, and demand vehicles [32] [33]. Wright [34] is the first to suggest the Gompertz model for biological growth. Perhaps the first to apply it to biological data was Davidson [35] in a hibovine body mass growth study. In 1931, the Gompertz model was reported by Weymouth, McMillin, and Rich [36] to successfully describe the shell-size growth in razor clams, as reported by Siliqua patula and Weymouth and Thompson [37] for the Pacific cockle, *Cardium Corbis*. Researchers quickly started to adapt the model to their data through regression. Over the years, the popular [38] model of Gompertz became a favorite regression model for many forms of organism growth, such as dinosaurs, e.g. [39] [40], birds, e.g. [17] [18] [30] [41], and mammals, e.g. [42] [43], including marsupials, e.g. [44] [45]. The Gompertz model is often also used to predict growth in microbe number or density [46] [47], tumor growth [21] [48] [49], and cancer patient survival [50].

Therefore, the purpose of this journal, the first is to describe the comparison of two forms of prediction models, that are logistic model and Gompertz model, in predicting the peak number of COVID-19 cases in Indonesia and discuss their usefulness. The second is to present and discuss the form of the logistic model and the Gompertz model. The results of this research are expected to provide benefits to (1) academics in Indonesia, (2) health practitioners in Indonesia, (3) industries in Indonesia, and (4) abroad.

The remainder of this journal is structured as follows: Section I explains the context and issue of this journal, the results of previous studies, the works related to the method similar to the method proposed, and presents the works proposed for this journal; section II describe the basic theory of logistic model and Gompertz model; section III describe the proposed works of this journal, detailed procedures, and the evaluation of the performance for logistic model and Gompertz model to forecast the peak number of COVID-19 cases in Indonesia; and the conclusions of this paper is described in section IV.

## 2. Basic Theory

### 2.1. Predictive Analytics

Predictive analytics is the process of data recognition of important and relevant trends. It draws from a range of different fields, some of which have been using data patterns for more than 100 years, including pattern recognition, statistics, machine learning, artificial intelligence, and data mining [51]. What distinguishes predictive analytics from other analytical types?

First, predictive analytics is data-driven, which means that algorithms derive key characteristics of the models from the data themselves rather than from the

analysts' assumptions [51]. Data-driven algorithms, to put it another way, induce models from data. The induction process may involve identifying variables to be used in the model, model-defining parameters, model weights or coefficients, or model complexity.

Second, algorithms for predictive analytics automate the process of identifying patterns from the data [51]. Effective induction algorithms discover the model's coefficients or weights and the model's very structure. For example, statisticians have created the logistic function called the sigmoid function to explain the properties of population growth in ecology, growing rapidly and maxing out at the carrying capacity of the environment. Other algorithms can be modified to forecast using the Gompertz model. The Gompertz model is well known and commonly used in many fields of biology. It has also been used to characterize plant and animal growth and the amount or volume of cancer cells and bacteria.

However, in this journal, the logistic model and Gompertz model application is modified, which is applied to predict the growth and the peak number of COVID-19 in Indonesia.

## 2.2. Theory of the Logistic Model

Logistic regression is a type of algorithm used for classification. This algorithm can be used to predict probabilities related to the occurrence of a class or event. Logistic regression is based on the logistic equation, which, as in probabilities, has output values in the range from null to one. Thus, the logistic function can be used to turn arbitrary values into probabilities. Logistic regression given any observed variable(s) gives the likelihood of an occurrence. Probability is often expressed as  $P(Y|X)$  and read as *Probability that the value is Y given the variable X*.

The logistic regression model can be expressed as follows

$$\ln\left(\frac{P}{1-P}\right) = m + kx. \quad (1)$$

Solving this equation for  $P$ , we get the logistic Probability

$$\frac{P}{1-P} = e^{m+kx}, \quad (2)$$

$$P = \frac{1}{1+e^{-(m+kx)}}. \quad (3)$$

We can, just as with linear regression, add several dimensions (dependent variables) to the problem

$$P = \frac{1}{1+e^{-(m+k_1x_1+k_2x_2+\dots+k_{N-1}x_{N-1}+k_Nx_N)}}. \quad (4)$$

In Eq. (4), the logistic model predicts the patient default *probability* based on input variables. The logistic model is carried out on a dataset not necessarily having a time component. Yet a "cross-section" of a time series data set (data were taken at a constant time point) is a strong candidate for a logistic model [52] [53]. Therefore, the logistic growth model improved the exponential growth model. A population grows in the logistic model until it achieves a maximum capacity. The logistic model is based on the assumption that the growth rate  $\frac{dy}{dt}$  is proportional to the existing population and the remaining available resources for the existing population. The concept of logistic growth is expressed in

$$y(t) = \frac{K}{1+e^{(-r(t-t_{inf}))}}, \quad (5)$$

where,

$$t_{inf} = \frac{1}{r} \ln \left( \frac{K}{Y_0} - 1 \right). \quad (6)$$

If  $y(t)$  represents a livestock's body weight at time  $t$ , then parameter  $K$  is in Eq. (5) could be considered a mature weight (the maximum weight that the livestock could obtain). Here  $t_{inf}$  is the time of inflection (the best time to expand a population) and  $r$  is the proportional growth rate parameter.

In another application of the logistic model, for example, in animal growth, the logistic growth curve is a model of three parameters which is typically given as

$$M = \frac{A}{1+e^{(-k(t-T_i))}}, \quad (7)$$

where  $M$  is body mass,  $A$  is asymptotic body mass,  $k$  is the constant growth rate,  $t$  is age and  $T_i$  is age at the point of inflection [54] [55] [56]. This model is rather inflexible, as around the point of inflection, it is symmetrical, fixed at 50 percent of the upper asymptote. The value of  $M$  at the point of inflection is  $t = T_i$ . Then the model reduces to  $\frac{A}{1+e^0} = \frac{A}{2}$ . Notwithstanding that limitation, one of the most popular and frequently applied models is the logistic model.

Another popular model for fitting time series data and forecasting is the Gompertz model and will be explained in the next subsection below.

### 2.3. Theory of Gompertz Model

The Gompertz model is well documented and commonly used in many biological aspects. It is also used to explain the growth of plants and animals and the amount or volume of bacteria and cancer cells [57]. However, in this journal, the Gompertz model will be applied to COVID-19 cases to predict the growth and the peak number of COVID-19 cases in Indonesia.

The growth curve for Gompertz is typically [54] [55] [56]:

$$M = Ae^{-e^{(-k(t-T_i))}}. \quad (8)$$

The Gompertz model is compared to the logistics model, which behaves very similarly. This is also inflexible in that 36.79 percent of the upper asymptote is fixed at the point of inflection. This can be found at  $t = T_i$ , reducing the model to  $Ae^{-1} = \frac{A}{e}$ . The Gompertz growth curve is, therefore, best suited for growth processes where the inflection point is located at approximately one-third of the adult value ( $A$ ). Perhaps even more common than the logistic model, the Gompertz model is fitted to chick growth data.

Some of the Gompertz model re-parametrizations found in the literature are more useful than others since they have simple parameters to interpret. One valuable re-parameterization that is commonly found is

$$W(t) = Ae^{-e^{(-k_G(t-T_i))}}, \quad (9)$$

where  $W(t)$  as a function of time (e.g., days from birth or hatching) is the expected value (mass or length), and  $t$  is the time,  $A$  is the upper asymptote (adult value),  $k_G$  is the growth coefficient (which affects the slope), and  $T_i$  is the inflection time. The

$T_i$ -parameter moves the growth curve horizontally without altering the shape. It is thus what is sometimes called a position parameter (whereas  $A$  and  $k_G$  are shape parameters).

The logistic model and Gompertz model are widely used to describe a population growth model. Both of these models will be applied to the COVID-19 case to predict Indonesia's growth and cases.

#### 2.4. Proposed Logistic Model

The logistic model used in the COVID-19 case in this journal has been modified. The modification here is to set the fitting coefficient, so it does not have an infinite value, to find the saturation point. But even so, the logistics model's function here still follows the basic function of the sigmoid curve. The logistic model for predicting the peak number of COVID-19 cases is as follows

$$Q_t = \frac{c}{1+e^{-(t-t_0)/a}}, \quad (10)$$

where  $Q_t$  is the cumulative value of the confirmed case,  $c$  is the maximum predicted value,  $a$  is the fitting coefficient, and  $t$  is the number of days since the first day the case appeared ( $t_0$ ).

#### 2.5. Proposed Gompertz Model

Same as the logistic model, the Gompertz model used in this journal for the COVID-19 case also underwent a slight modification to the function of the fitting coefficient, with the following equation

$$Q_t = ce^{-e^{-(t-t_0)/a}}, \quad (11)$$

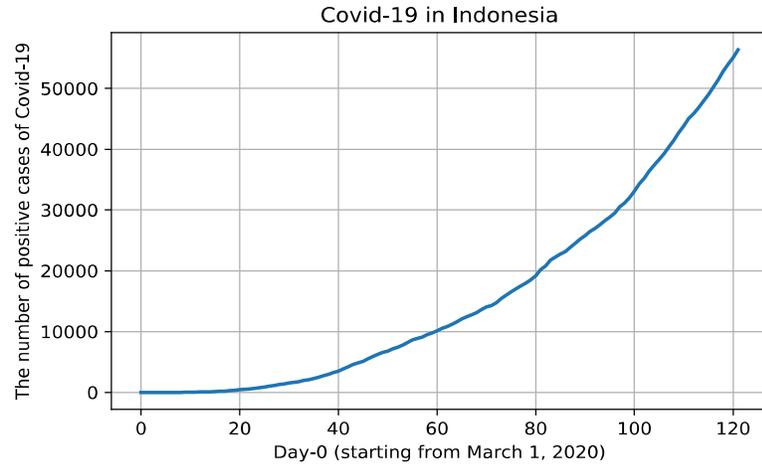
where  $Q_t$  is the cumulative value of the confirmed case,  $c$  is the maximum predicted value,  $a$  is the fitting coefficient, and  $t$  is the number of days since the first day the case appeared ( $t_0$ ). Eq. (10) and Eq. (11) are based on the fitting-curve.

The next section will discuss the performance evaluation of the time series prediction model using the logistic model and the Gompertz model.

### 3. Performances Evaluation

#### 3.1. The Number of Cases of COVID-19 in Indonesia

Figure 1 shows COVID-19 cases in Indonesia, starting from March 1, 2020, until June 30, 2020. It shows that the curve is increasing from 0 (March 1, 2020) to 56385 (June 30, 2020), which shows the number of people that are positively infected by COVID-19. For example, on day-0 there are 0 people, on day-20 there are 450 people, on day-40 there are 3512 people, on day-60 there are 10118 people, on day-80 there are 19189 people, on day-100 there are 33076 people, and on day-121 (June 30, 2020) there are 56385 people that positively infected by COVID-19.



**Figure 1** The Number of Infected People in Indonesia

### 3.2. The Increment of COVID-19 in Indonesia

This journal's history of data is starting from March 1, 2020, and ends on June 30, 2020. Figure 2 concluded that the increment of cases of COVID-19 is known as the delta ( $\Delta$ ). This means the cumulative (*cum*) between the number of infected people in this day ( $cum(i)$ ) minus the number of infected people yesterday ( $cum(i - 1)$ ), or it can be expressed as

$$\Delta(i) = cum(i) - cum(i - 1), \tag{12}$$

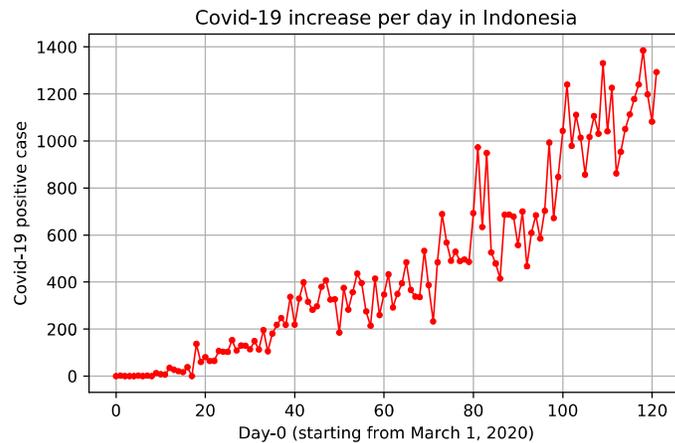
or

$$\Delta(i + 1) = cum(i + 1) - cum(i), \tag{13}$$

where

$$\Delta(0) = cum(0), \tag{14}$$

which mean the  $\Delta(0)$  indicated the day-0 as known as the initial cases, at March 1, 2020, with the number of cases is 0.



**Figure 2** The Increment of Infected People per Day in Indonesia

Figure 2 shows the increment of the number of cases of COVID-19 per day, starting from March 1, 2020, until June 30, 2020. This time series shows some distinct seasonal patterns. In general, Figure 2 shows deltas always rising, but there

are decreases and increases in detail from one day to another. As shown in the figure, on day-69 to day-71, there is a two-fold decrease from 533 to 387 to 233 number of people. But after that, there was a threefold increase, from 233 to 484 then to 689 people. Also, on day-96 to day-97, the increase is significantly higher, 993 people.

Based on the pattern, the conclusion is that the number of infected people day by day in Indonesia is up and down continues to increase, and has experienced a significant decrease, and vice versa. Since the seasonal pattern is not exactly replicated every day, there will always be some cyclic and erratic behavior. Do note the long-term pattern, illustrated by the overall increase over time (except, maybe, towards the end of the series).

### 3.3. The Comparison among The Logistic Model, The Gompertz Model, and The Real Data

The result shows that Figure 3 can be concluded that the starting cases (initial value) on March 1, 2020, based on the logistic model, there are 884 infected people. Furthermore, there are 370 infected people based on the Gompertz model. However, based on the fact on Indonesian media, it is known that on March 1, 2020, there are 0 cases of infected people. Also, there is a difference in initial values (day-0) between the logistic model, the Gompertz model, and the real data, as shown in Figure 3, due to a prediction's characteristics. Predictions are initially messy, and the similarity of data is low, but the farther here, the predictions for the future will be more accurate, as shown in Figure 3. The longer here (which means, the more data learned), the similarity of data between the prediction model and the reality is higher (relatively more similar). There is a significant difference only in the initial values, but overall, the predicted data and the real data are almost identical. Also, we look at the accuracy of these predictions based on the data's similarity, not seen in detail from day to day.

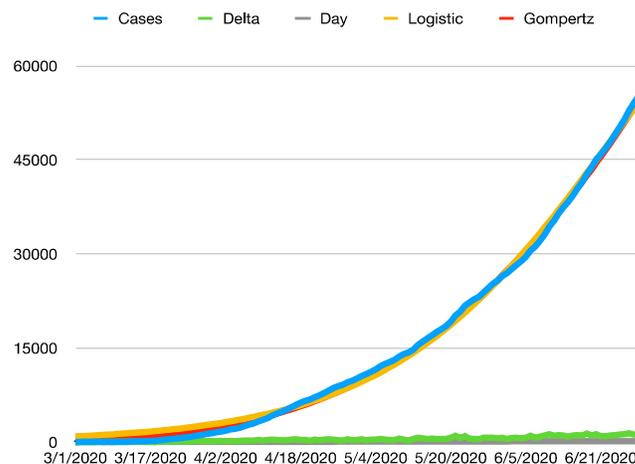
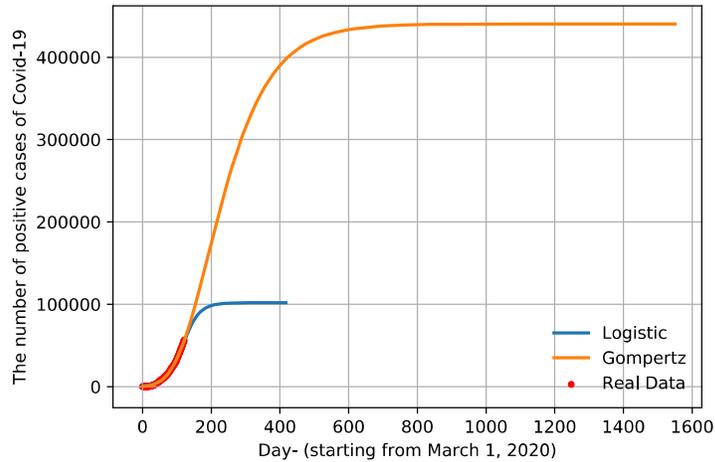


Figure 3 The Comparison of Data

### 3.4. The Predicted Number of Cases

The predicted number of cases is conditionally based on the history of data used. In this journal, the length of the history of data used is 121. Afterward, as many as 121 of these historical data are learned by machine learning. The prediction of the peak COVID-19 is made, including the peak day. Based on the research of this journal, for example, the history of data taken by this journal is until June 30, 2020,

then the saturation point indicated that the COVID-19 would end, is on April 23, 2021, for logistic model, and will end on May 30, 2024, for Gompertz model. However, if the history of data used is until June 6, 2020, the saturation point that indicated that the COVID-19 would end is on December 31, 2020, for the logistic model, and will end on June 4, 2022, for the Gompertz model.



**Figure 4** The Forecasting of Infected People in Indonesia

The predicted number of infected people uses the logistic model is shown in Figure 4. This figure shows the increment of infected people from the beginning to the end (day-417), with the peak number of infected people is 101856. A similar calculation applies for the Gompertz model in Figure 4 that shows the predicted number of infected people from the beginning to the end (day-1550). The peak number of infected people is 440405. Figure 4 represented the number of infected people, where the history of data is started on March 1, 2020 (day-0) and ended on June 30, 2020 (day-121).

The blue line represents the predicted line obtained by these models and the orange line for the logistic model and Gompertz model. The objective of these predictions is to know about the ending of the COVID-19 outbreak. Therefore, these results can be used by all elements to prepare to face this outbreak. From Figure 4, we can conclude that the saturation number occurs on day-392 to day-417 and day-1436 to day-1550 for the logistic model and Gompertz model, respectively. The saturation number indicated no addition of infected people, which means the outbreak is stuck at a saturated value, or we can conclude that the outbreak is over.

Subsequently, to find out the accuracy of each model (logistic and Gompertz), then the  $R^2$  score parameter is used by the following equation:

$$R^2 = 1 - \frac{\sum(y_i - \hat{y}_i)^2}{\sum(y_i - \bar{y})^2} \tag{15}$$

$$R^2 = 1 - \frac{g}{z} \tag{16}$$

$R^2$  score range from 0 to 100%. It is closely related but not the same as the mean square error (MSE).  $R^2$  is the proportion of variance that is expected from the independent variable(s) in the dependent variable. Another definition is (total variance explained by the model) / total variance. Thus, if it is 100%, the two variables are completely correlated, i.e., without any variation at all. A low value will indicate a low degree of correlation, indicating a regression model that is not flawed but is not accurate in every situation.

Based on Eq. (15) and Eq. (16), we perform this calculation in three steps to make it easier to understand.  $g$  is the sum of the differences between the values observed and those predicted  $(y_{test_i} - pred_{s_i})^2$ , or in another definition  $g$  is the total sum of residuals.  $z$  is each observed value  $y_i$  minus the average of observed values  $np.mean(y_{test})$  or the total sum of squares. If explained in more detail, then  $y$  is the original value with index  $i$  ( $y_i$ ),  $\hat{y}$  is the predicted value with index  $i$  ( $\hat{y}_i$ ), and  $\bar{y}$  is the value average.

#### 4. Conclusions

According to the logistic model, with an accuracy of 0.9966 (99.66%), the peak number of cases in Indonesia is 101857 people, and the peak of the outbreak is 418 days after March 1, 2020, is on April 23, 2021. Simultaneously, according to the Gompertz model, with an accuracy of 0.9985 (99.85%), the peak number of cases in Indonesia is 440406 people. The outbreak's peak is 1551 days after March 1, 2020, is on May 30, 2024. These models use the history of data from March 1, 2020, until June 30, 2020.

Based on this journal's results, we can conclude that the saturation point of the predicted number for the logistic model and the Gompertz model is conditionally based on the length of history of data. Therefore, the more history of data used, the accuracy of the predicted value will be more accurate, because the machine will be more sophisticated in learning the patterns that will be used to predict the future. Also, the principle of machine learning is that the more data learned by machine learning, the more accurate the predicted data.

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