



Load Balanced User Association Scheme in Planned Non-bias Small Cell Deployment

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ARTICLE INFO

Received May 11th, 2022
Revised September 2nd, 2022
Accepted September 3rd, 2022
Available online January 25th, 2023

Keywords

heterogeneous networks, radio resource optimization, load balancing, efficient user association

ABSTRACT

The work presented in this paper focuses on developing a load-balanced user association scheme in a single Radio Access Technology (RAT) Heterogeneous Networks (HetNet). The optimization algorithm maximizes network capacity and optimal user association in a planned small cell deployment framework, considering Quality of Service (QoS) and network load balance. Planned deployment of small cells has many advantages, including controlled interference levels within the HetNet. The proposed Branch and Bound (BAB) based algorithms provide a more realistic interference scenario through the non-use of bias in the small cells. The proposed algorithm achieved higher load balancing than current schemes in literature and high spectral efficiency values.

Acknowledgment

The authors are grateful to all those who contributed to the study by sharing their experiences and knowledge. We acknowledge Multimedia University - Telekom University Joint Research Grant 2021 for providing financial sponsorship to facilitate this research project (Project Code: MMUE/210067).

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<https://doi.org/10.25124/ijait.v6i01.4835>

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1. Introduction

Offloading within the same technology or in-band offloading deploy small cells that use the time-frequency resources as the macro cells. The advantage of this type of offloading is increased spectrum efficiency, reduced signaling overhead, shorter delay for non-delay tolerant data, and seamless mobility management [1, 2]. Offloading algorithms are designed to assign users to either the macro or small cells. These algorithms are either controlled in a centralized fashion at the macrocell or in a distributed manner at the User Equipment (UE). These algorithms are known as Radio Resource Management (RRM) or user association algorithms. They are assessed using metrics like but not limited to network topology, handoff, mobility, load priority cell association, network load balancing, back-haul bottleneck, and interference management. In general, single criteria algorithms have implementation simplicity, but they may lead to unbalanced load distribution, high call blocking probability, or may not guarantee Quality of Service (QoS). Multiple criteria algorithms using utility functions and computational intelligence or fuzzy logic are more efficient but may be difficult to implement [2]. The higher transmit power of the macrocell in a HetNet causes a significant disadvantage to small cells. The users do not prefer associating themselves with the small cell because it provides less SINR gain and, thus, less capacity. This situation leads to the under usage of small cells and creates redundancy in small cell deployment. A cell bias was added to the transmit power of the small cell to reduce transmit power imbalances and induce more user attachment to small cells to achieve better load balancing in a multiple-tier HetNet [3–6]. While much research is contributed toward finding the optimal bias value, past work has noted that prescribing the optimal bias value is tedious and complex [7]. The use of an artificial bias may increase the interference to the user who attaches to the small cell, due to the user's proximity to the macrocell. This leads to the argument that using a bias may not solve the issue of optimal user association to a small cell in a HetNet.

The three main factors that motivate cell planning optimization are interference, user location, and radio propagation. Unplanned cell deployments may lead to localized improvements but deteriorate overall network performance by adding to interference [8]. The distribution of the users also plays a role because this can affect the load balancing between the macro and small cells. The study by [9] highlight the basic aims for low-powered node deployment are reducing cross-tier interference, and maximizing overall HetNet capacity, adaptability, and low complexity self-organizing features. Deploying small cells in an unplanned manner may cause unpredictable interference and leave unwanted gaps in coverage. Due to the lower antenna heights, small cells have to overcome propagation losses due to the Non-Line of Sight (NLOS) conditions.

Furthermore, one small cell may have Line of Sight (LoS) conditions to multiple macrocells. This makes channel modeling for small cell propagation very challenging. The study in [10] highlighted that using fixed locations for small cells or base stations is advisable since modeling BS as random locations in a HetNet does not precisely control association region and load distribution. A high small cell to macrocell ratio leads to inefficient spectrum utilization for a given coverage area. It may provide a non-optimum capacity, and proper spatial separation is critical to control the interference in Heterogeneous Networks (Het-Net) [8, 11]. The work by [12] highlights that the simple computational analysis of Signal to Interference plus Noise Ratio (SINR) under the grid topology deployment is still highly desirable in network planning. A fixed deployment model is also proposed by [13], where the macro BS is overlaid by uniformly and independently deployed

picocells. This work proposes a fixed small cell deployment framework, where the small cells are deployed at a fixed distance from the macrocell in a grid fashion. In this work, a non-bias based QoS controlled RRM Scheme that maximizes HetNet capacity with high network utilization, spectral efficiency, and load balancing is therefore proposed in this work. The scheme uses a Branch and Bound algorithm that performs the user association and subband allocation. A small cell deployment framework that proposes the deployment of small cells closer to the macrocell for network capacity enhancement, better load balancing, and higher spectral efficiency is also proposed.

2. System Modelling and Algorithms

2.1. Branch and Bound Algorithm

The RRM uses a Branch and Bound (BAB) algorithm to perform the maximization of the capacity. This algorithm has been used in recent RRM studies [14–16]. BAB is a non-heuristic, global optimization algorithm used in nonconvex problems. The algorithm either searches all leaves in the tree or prunes off infeasible branches, allowing the algorithm to provide an optimal solution. BAB algorithms can converge quickly in certain scenarios. The BAB algorithm uses a complete, ordered listing of all the elements in a set known as an enumeration tree, which is illustrated in Figure 1. As shown in Figure 1, the algorithm splits the parent set into subsets and begins the search at the top of the tree. The parent set, $S(a)$, with A elements, is given as [14]

$$S(a) = \{1, 2, \dots, a, \dots, A\} \tag{Equation 1}$$

$$J(s(a)) = \max_{x(a) \subseteq S(a)} Js(a) \tag{Equation 2}$$

All nodes at the next level are analyzed, and the largest criterion function, J , is determined. The largest criterion function, J , is expressed using Equation 2, where $s(a)$ is the global maximum optimal subset value. L denotes the lower bound. The incumbent best subset is determined if $J \geq L$, the lower bound is updated with the new J , and its successor nodes are further explored. If $J \leq L$ for a node, the successor nodes are pruned because they cannot be the optimal subset given the monotonicity of J . Monotonicity indicates that the subsequent searches are either entirely non-decreasing or non-increasing.

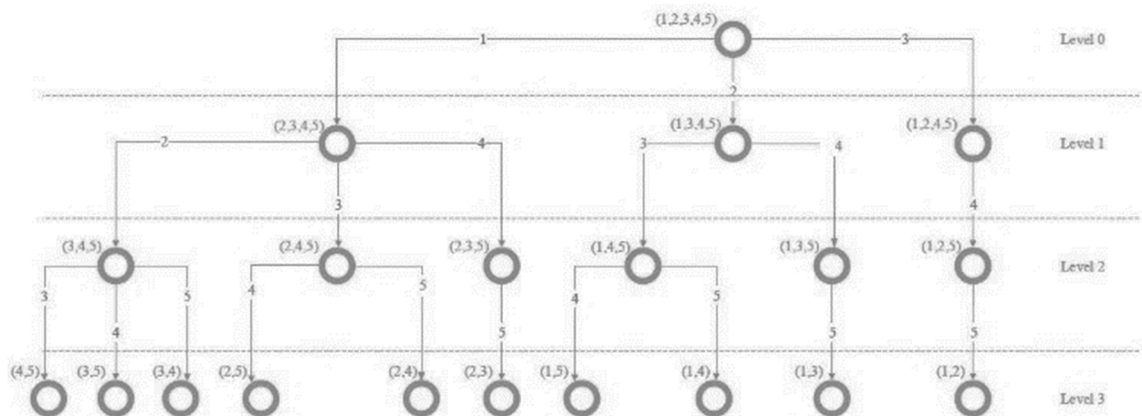


Figure 1 Enumeration Scheme of Branch and Bound Algorithm [17]

2.2. Proposed BAB-based Algorithms and Optimization Modelling

The user association variable x_{ukf} is generated when Equation 3 is satisfied, x_{ukf} is created for the desired subband, $f=1, 2, \dots, F$. x_{ukf} is a subset of x_{optim} where x_{optim} is the set of x_{ukf} maximized HetNet capacity.

$$p_k/L_{uk} \geq \alpha \quad \text{Equation 3}$$

where p_{tk} is the node transmit power and L_{uk} is either pathloss of the macrocell or the pathloss of the small cell. The α value is the LTE receiver sensitivity value of -103.530dBm for Quadrature Phase Shift Keying (QPSK) 1/8 modulation.

Three algorithms are tested, constrained through minimum receive power, minimum subband allocation, and minimum user throughput. The objective of the three algorithms is to maximize the network capacity in the Heterogeneous Networks (HetNet) through the objective function

$$\text{maximize } z = \sum_{u=1}^U \sum_{k=1}^K \sum_{f=1}^F B_f \log_2(1 + SINR_{ukf}) \quad \text{Equation 4}$$

The first algorithm, the Branch and Bound No Subband Constraint User Association (BABRSS) algorithm, use a Received Signal Strength (RSS) based user association scheme. They constrain user association to a single base station association through Equations 5 and 6 since single cell attachment ensures better load balance than multiple cell attachment policies.

$$\sum_k y_{uk} = 1 \quad \forall u \in \{1, 2, \dots, U\} \quad \text{Equation 5}$$

$$\sum_f x_{ukf} \leq y_{uk} M \quad \forall u \in \{1, 2, \dots, U\}, \forall k \in \{1, 2, \dots, K\} \quad \text{Equation 6}$$

Where M is any arbitrarily large number as long as $M > F$, in this work, M is set as 1000. y_{uk} is a binary variable used to control the user association. To ensure no subband is allocated to two users in a single timeslot, the inequality constraint expressed by Equation 7 is used. It should be highlighted that the number of subbands that user u is attached to at any given timeslot is not limited to one.

$$\sum_u x_{ukf} \leq 1 \quad \forall k \in \{1, 2, \dots, K\}, \forall f \in \{1, 2, \dots, F\} \quad \text{Equation 7}$$

The second algorithm, Branch and Bound Minimum Subband Constraint User Association (BABMS), forces each newly added user to be allocated a subband without any minimum QoS constraints. Along with the constraints in Eq. 5, Eq. 6, and Eq. 7, BABMS introduces a new constraint, where at least one mandatory subband must be allocated to a user. This constraint is controlled by:

$$\sum_k \sum_f x_{ukf} \geq 1 \quad \forall u \in \{1, 2, \dots, U\} \quad \text{Equation 8}$$

Equation 8 prescribes that each user can only be attached to one cell and must be allocated at least one subband. The third algorithm, Branch and Bound Minimum User Throughput User Association (BABMTP), introduces an alternative constraint to Equation 8. In the new constraint, Equation 9 is a non-linear constraint that disallows user attachment to a cell if the minimum throughput, r_{ukf} , is not met.

$$\sum_k \sum_f B_f \log_2(1 + SINR_{ukf}) \geq r_{ukf} \quad \forall u \in \{1, 2, \dots, U\} \quad \text{Equation 9}$$

2.3. Simulation Settings

Table 1 shows a summary of all simulation parameters. The assumption made in this simulation is that all users are simultaneously downloading and therefore have fair access to the network resources, based on the user's distance from the cell, d_{uk} . The distance affects the minimum user sensitivity receivable by the user. The transmit antennas of the small cells and the macrocell are assumed as omnidirectional. Rayleigh fading is also assumed as the channel condition in the simulation.

Table 1 Simulation Parameter

Parameter	Units	Macro Cell (LTE)	Small Cell (LTE)
Frequency band	GHz		2.6
Max EIRP	dBm	46	30
Cell Radius	m	500	50
Distance from macrocell	m	-	50
Small cell- small cell ISD	m	-	100-111
Small cell- macrocell ISD	m	50	-
Receiver Gain	dB		1
Noise Figure	dB		9
Max Users in HetNet	-		24

2.4. HetNet and User Distribution Modelling

The single Radio Access Technology (RAT) HetNet consists of a Long Term Evolution (LTE) macrocell with a radius of 500m and is overlaid with between 1-4 LTE small cells situated at varying ISD distances of 50m, 75m, 100m and 125m from the macrocell center. K denotes the total number of base stations in the HetNet. The small cells have a radius of 50m and transmit at 30dBm. The macrocell transmits power is 46dBm [18]. The simulation is done in a $1 \times 1 \text{ km}^2$ area, with the macrocell and small cells deployed in a hexagon grid layout. The small cells are progressively added in a planned manner until a maximum of 4 small cells are deployed. Figure 2 shows how the cells are deployed in a planned manner and located 50m from the macrocell. The small cell distance from macrocell and the number of small cells are varied to study their effect on the users' performance in the HetNet. This work uses 24 users and 8 subbands per cell in the simulations. The proposed model can be used to optimize any number of K , U , or F , with a restriction that F must be increased in the form of 2^{γ} where γ is an integer.

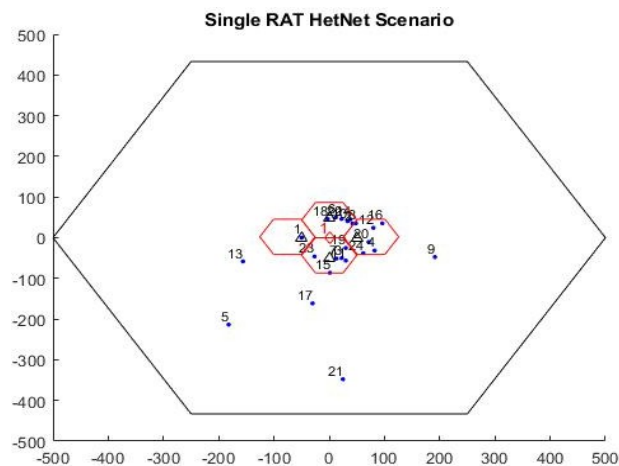


Figure 2 HetNet with four small cells deployed in a planned manner at 50m distance from macrocell

The users were generated based on non-uniform distribution, where users are partly clustered around pico cells [18]. All results are averaged from 10 different user sets, U_s , where U_s , $s = 1, 2, \dots, 10$. These user sets are manually generated and then fed into the algorithm. The maximum number of users, U , in each user set is 24. For each user set, the performance metrics are analyzed by varying the number and location of small cells deployed inside the macrocell coverage area. The results shown are averaged over all the user sets. Once a user-set is generated, it is used consistently while the location and quantity of small cells inside the macrocell coverage area are manipulated. The user location is fixed to allow the improvement in terms of data rate for selected users to be studied. After the maximum four small cells and varying distances are reached, the user-set is changed, and the process of varying the small cell quantity and location is repeated. The users are assumed to be streaming data and are non-mobile.

2.5. Pathloss Modelling

The path loss modeling factors in both shadowing loss and Rayleigh fading. The macrocell and small cell path loss use the models from [18] and are calculated using:

$$L_{macro} = 128.1 + 37.6 \log_{10} d_{uk} \quad \text{Equation 10}$$

$$L_{smallcell} = 140.7 + 37.6 \log_{10} d_{uk} \quad \text{Equation 11}$$

Where d_{uk} is the distance between cell k and user u in km. L_{macro} and $L_{smallcell}$ are the path loss in dB for the macrocell and small cell, respectively, and an assumption of 10dB for the standard deviation of shadow fading is made.

2.6. Capacity and User Throughput Modelling

Intercellular interference is used to calculate the SINR and is considered only if the same RE f is reused by two different cells (macrocell or small cell) in the immediate adjacent tier. The SINR of desired user u , when served by cell k and using subband f , is expressed as [19].

$$SINR_{ukf} = \frac{x_{ukf} P_{ukf} G_{ukf}}{\sum_{\substack{i=1 \\ i \neq u}}^U \sum_{\substack{j=1 \\ j \neq k}}^K x_{ijf} P_{ijf} G_{ijf} + n(t)} \quad \text{Equation 12}$$

where x_{ukf} is the user association variable when the desired user u is connected to cell k on subband f . P_{ukf} is the transmit power of cell k to user u , and G_{ukf} denotes link gain from cell k to user u over desired subband f . x_{ijf} denotes the user association variable for other users who use subband f on other cells. x_{ijf} is associated with received power P_{ijf} and link gain G_{ijf} . The throughput of the desired user u , r_{ukf} is calculated using:

$$\sum_k^K \sum_f^F B_f \log_2(1 + SINR_{ukf}) = r_{ukf} \quad \forall u \in \{1, 2, \dots, U\} \quad \text{Equation 13}$$

The mean throughput for all users in the HetNet, τ_{HetNet} is then calculated using

$$\tau_{HetNet} = \frac{\sum_u^U r_{ukf}}{U} \quad \text{Equation 14}$$

2.7. Performance Metrics

Apart from the throughput, the spectral efficiency η_{HetNet} and Jain's fairness index, Λ [7,20] were used to measure the effectiveness of the algorithms proposed. The Jain's fairness index is calculated using

$$\Lambda = \frac{(\sum_{k=1}^K \rho_k)^2}{K \sum_{k=1}^K (\rho_k)^2} \quad \text{Equation 15}$$

Where K is the total number of base stations in the HetNet, the maximum fairness index is 1, and the larger the value of FI, the more balanced the system's distribution is. The spectral efficiency, η_{HetNet} , assuming that 20% overhead is used in the channel, is calculated using

$$\eta_{HetNet} = \frac{DR}{B} * 0.8 \quad \text{Equation 16}$$

Where DR is the data rate of a channel in bps, B is the channel bandwidth in Hz.

3. Results and Performance Analysis

The results focus on the throughput performance of selected users to assess the performance of the algorithms. User1 is a user located outside of small cell coverage but approximately 50m from macrocell. User2, on the other hand, is a user located at the edge of small cell coverage. User3 is defined as a user located inside small cell coverage. The locations of User1, User1, and User3 are kept constant throughout each user set while their peer's locations vary. This is to enable the study of the performance of these users under varying peer user distribution.

The results in Figure 3 show that without an offload constraint, the throughput of macro-only users close to macrocell center increases as the number of small cells increases when using BABRSS and BABMTP. The BABMTP outperforms the BABRSS by 2% in terms of User1's throughput performance during provide Quality of Service (QoS) control.

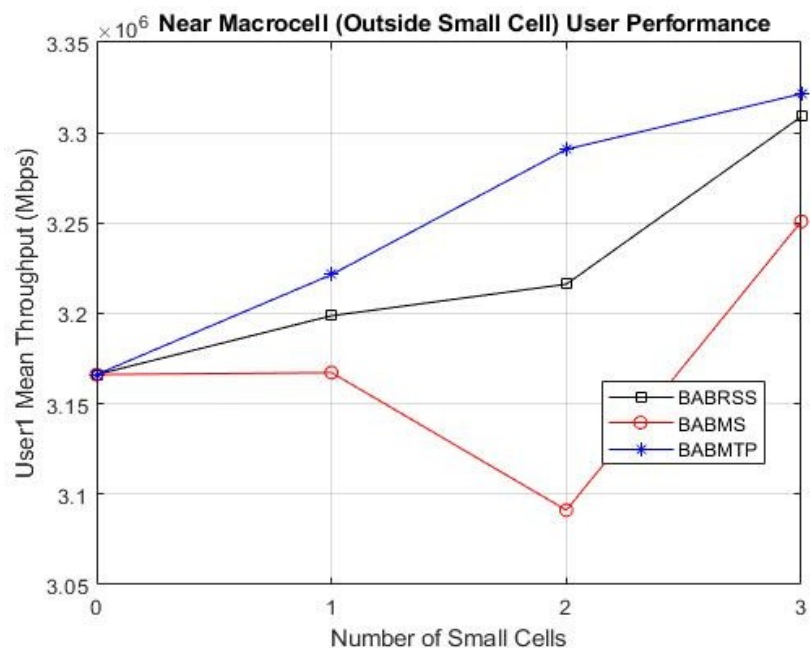


Figure 3 User1 Mean Throughput Performance for Different Small Cell Numbers

User2's performance using the different schemes is further shown in Figure 4.

The colored boxes show the user association to the macro- cell and small cell. For BABRSS and BABMS, User2 is associated with small cells until K=3. At K=4, User2 is re-associated to the macro-cell as other macro-only users are offloaded to other small cells. The BAB algorithm forces the re-association to the macrocell as this association will contribute to the capacity maximization of the HetNet. The results seen for BABMTP are consistent with the previous analysis, whereby User2's throughput is more stable while connected to this scheme. User2 can also get a higher throughput when consistently connected to the small cell, even as the number of small cells is increased. Edge users of small cells can be allocated to any adjacent small cell as the optimization process strives to maximize the HetNet capacity. BABMS' ability to provide higher throughput for User2 comes at the expense of being toggled between cells to another small cell. BABMS requires a minimum subband allocation as compared to BABMTP, which requires a minimum throughput allocation. By implementing a minimum throughput constraint, BABMTP can provide a more stable throughput with less user reconnection to adjacent cells.

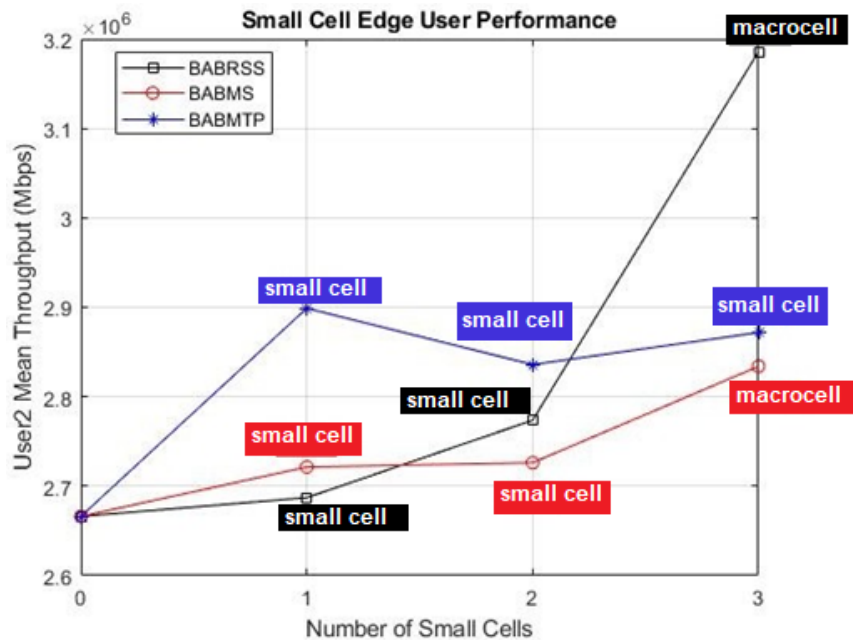


Figure 4 User2 Mean Throughput Performance for Different Small Cell Numbers

The average user throughput of User3 can be seen in Figure 5, and the user association to the macrocell and small cell is shown in the colored boxes in the figure. The increase in User3's throughput is significant when connected to the small cell, with the BABMTP scheme achieving the highest user throughput. This shows that through BABMTP, users attached to the small cell, even when the user is near the macrocell, can achieve significant improvements in user throughput.

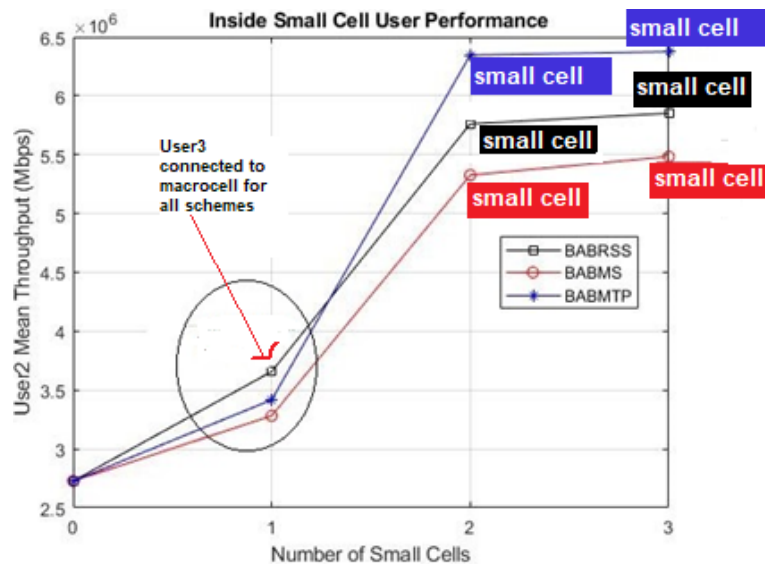
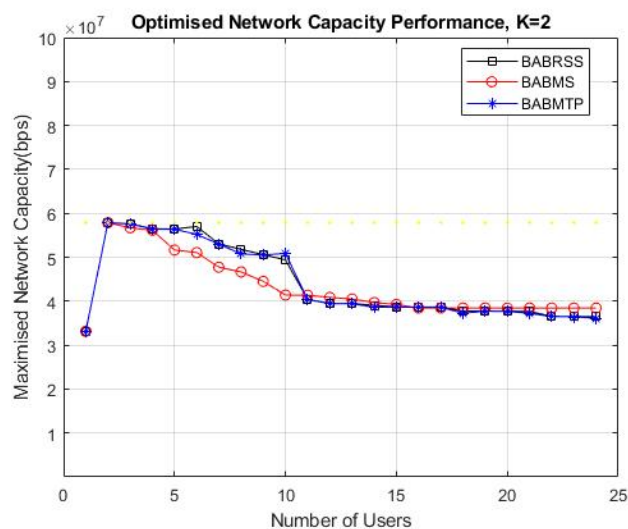
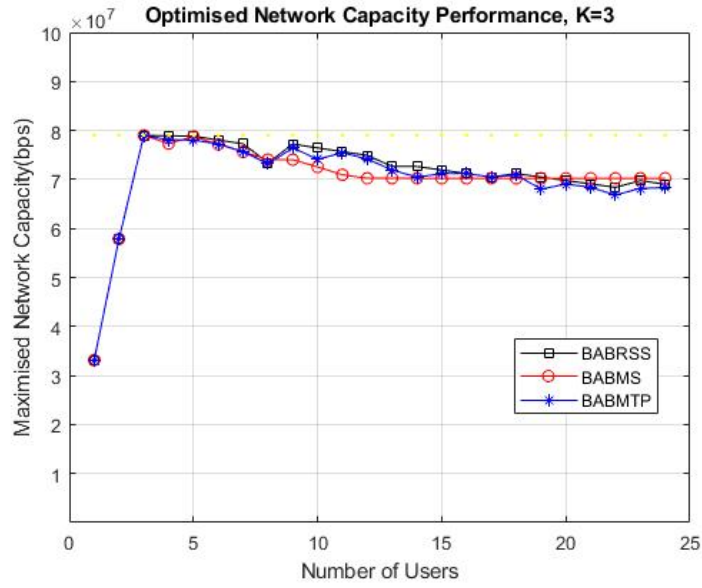


Figure 5 User3 Mean Throughput Performance for Different Small Cell Numbers

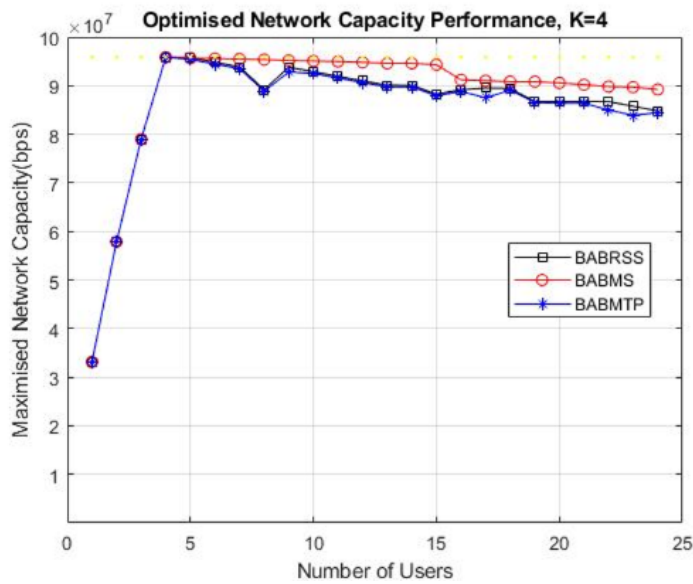
Figures 6 (a) – 6 (c) show the maximized HetNet capacity achieved from the three schemes. The yellow dotted line on these figures benchmarks the maximum theoretical capacity available as the K increases. It was observed that the BABRSS and BABMTP are almost similar as the number of cells increases. As the number of cells increases, the available theoretical capacity in the HetNet also increases. These results show that despite the constraint that guarantees the QoS for a user connection, BABMTP performs in the same greedy manner as BABRSS performs, which is to maximize the network capacity. Based on Figure 6 (c), BABMS performs the best when $K=4$ and appears to outperform the other two schemes in terms of maximizing the capacity. However, it should be highlighted that BABMS does not concern itself with the user connection QoS and is only concerned with ensuring each user is provided at least the minimum subband allocation. The predicted spectral efficiency for $K=2, 3,$ and 4 are 2.893 bits/s/Hz, 3.9514 bits/s/Hz, and 4.7932 bits/s/Hz are summarized in Table 3, showing that the BABMS scheme has the highest spectral efficiency for all K . The BABRSS scheme performs marginally better than the BABMTP scheme, with a higher user drop rate tradeoff. A fairness index computation was done to estimate how the load balancing is achieved in the HetNet. The fairness achieved is shown in Table 3.



(a)



(b)



(c)

Figure 6 (a) – 6 (c) Maximized HetNet capacity achieved from the three schemes.

The results show that the highest fairness is achieved when three small cells are used in the HetNet. The BABRSS shows decreasing fairness as the number of cells is increased. The BABMMS scheme achieves the highest fairness when two small cells are deployed. The BABMTP shows an increasing trend in load balancing fairness as the number of cells increases, with the highest being 0.9751 at three small cells. The proposed BABMTP scheme is compared with some schemes in the literature, as shown in Table 2.

Table 2 Optimized Load, Spectral Efficiency and Fairness Index

Total Cells in HetNet, K	2	3	4	
Predicted load (Mbps)	57.860174	79.028900	95.864568	
Optimized load, DR_{actual} (Mbps)	BABRSS	36.473013	69.023872	84.799233
	BABMS	38.437236	70.237274	89.210031
	BABMTP	36.108283	68.319829	84.460036
Spectral efficiency Based on Predicted Load (bits/s/Hz)	2.893	3.9514	4.7932	

Spectral Efficiency (After Optimization) (bits/s/Hz)	BABRSS	1.837	3.4512	4.2400
	BABMS	1.9219	3.5119	4.4605
	BABMTP	1.805	3.4160	4.2230
Fairness Index (Δ)	BABRSS	0.9687	0.9618	0.9573
	BABMS	0.9772	0.9793	0.9457
	BABMTP	0.9570	0.9591	0.9751

To make a fair comparison, the scenarios were simulated with 30 and 40 users and four small cells. The positions of the small cells were not changed, and again 10 user sets for 30 and 40 users were used to get an average result. Results in Table 3 show that two schemes in literature that use predetermined bias values [4,5] have reported lower load balancing fairness values than the proposed BABMTP scheme. Both works use a random pico deployment. Furthermore, the system throughput reported by [5] is lower at 53Mbps, as compared to the BABMTP scheme, which achieved 98.33 Mbps. The work by [20] does not use bias for user association. Instead, it uses a gain prioritized awareness scheme. The work by [20] achieved a higher mean user throughput than the BABMTP scheme. However, the proposed BABMTP scheme achieved a much higher load balancing fairness of 0.9622 as compared to the 0.925 achieved by [20].

Table 3 Comparison of performance of proposed BABMTP scheme

	[4]	[5]	[20]	BABMTP		
Users	40	30	30	24	30	40
Total Cells	4	4	4	3	4	4
Pico cell deployment	Random	Random	Not specified	Fixed 50m ISD		
Bias	3,6,9,12,15 dB	4dB	None	None		
Method	Dynamic Load Balancing	Gain Access Aware	Gain Prioritized Aware	BABMTP		
System throughput (Mbps)	53	-	-	108.23	109.64	98.33
User throughput (Mbps)	1.325	-	8.7-9.3	3.54	3.65	2.46
Load balancing fairness index	0.85	<0.95	0.925	0.98	0.962	0.964

The comparison shows that the BABMTP scheme can achieve a load-balanced user association scheme with QoS control that is more efficient than existing schemes in the literature. The non-use of bias has reduced the steps involved in the user association process, as the optimal value of the bias does not need to be determined. Furthermore, the non-bias use does not create an artificial SINR scenario for the user, thus ensuring the advertised throughput rate is the actual achievable rate when the user attaches himself to the small cell.

4. Conclusion

Using bias for the small cells to attract users to offload from macro cells can sometimes be complicated, as estimating the optimal bias value can be time-consuming. Bias-based user association also does not guarantee the advertised throughput to the user once attached to the small cell. As such, non-bias-based user attachment algorithms should be used in HetNets. BAB algorithms are efficient as they provide an efficient solution to the user attachment problem. Three BAB-based algorithms were based on base station attachment constraints (BABRSS), based on minimum subband (BABMS), and based on minimum user throughput (BABMTP). They were analyzed based on selected user throughput performance in a HetNet network comprising macro and small cells in an outdoor environment.

The small cells were deployed using a planned framework whereby the small cells were positioned at fixed distances from the center of the macrocell. The non-use of small cell biasing provided a more realistic scenario of the SINR experienced in the HetNet. All algorithms controlled the user association to a given cell via a minimum power threshold. The optimization aimed to maximize the HetNet capacity while ensuring user and HetNet QoS. The BABMTP scheme showed high spectral efficiency and an increasing trend in the load balancing fairness index as the number of small cells increased. Comparing the BABMTP to existing schemes in the literature showed superior load balancing fairness and system throughput achievement.

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